# INDUCTIVE LOGIC

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# PREFACE

It has been my aim, in the following pages, to present the essential features of inductive logic, in the hope that this work may prove a fitting supplement to the elementary courses in formal or deductive logic. The impression is too often left in the minds of those who have pursued the study of deductive logic exclusively that the formal laws of the syllogism constitute the entire body of logical doctrine, and that reasoning consists solely in drawing conclusions from given premises. There is danger here lest reasoning become associated with an artificial procedure that seems to find its proper sphere in the solution of verbal quibbles and logical puzzles. In the actual experiences of life, we do not find our premises ready made. They are the result of wide observation and patient investigation and experiment. We challenge premises that are given, and weigh their significance. We meet particular facts before we do the general laws.

The former must be tested and interpreted, before we can rise to the general laws which underlie them, and which may stand as the major premises of our syllogisms. Thus within the very sphere of deduction itself there naturally opens a wide field for inductive inquiry. Therefore I have emphasized the necessity of a thorough knowledge of the principles of inductive logic in order to comprehend the material as well as the formal elements in inference, and without which no firm grasp of the general process of reasoning is possible. I have also insisted upon regarding induction and deduction as mutually dependent; not as separate modes of inference, but rather as different phases of one and the same logical procedure.

I have endeavored, also, to indicate in some measure at least, the salient characteristics of the modern logic, especially as presented in the works of Lotze, Sigwart, Jevons, Green, Bosanquet, and Venn. In the illustrations of the various inductive methods I have sought fresh material as far as possible, with the view of representing the actual modes of reasoning and methods of investigation employed by those who have become eminent in their several spheres of research, such as Faraday, Tyndall, Darwin, and

Lubbock; and especially the different methods which have led to important discoveries in the various sciences. This applies not only to the illustrations in the text proper, but also to those which I have collected in Chapter XX. under the head of Logical Exercises. It seems to me, moreover, that inasmuch as the principles of inductive investigation are in such accord with the scientific spirit of our age, their importance as a logical discipline cannot be too highly valued.

J. G. H.

PRINCETON, N.J., March 2, 1896.

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## CHAPTER I

### The Nature of Inference

Induction is a particular mode of inference in general; and therefore before its nature and scope can be adequately defined, it will be necessary to give some account of the theory of inference, and its precise logical signification. Moreover, it is not possible to appreciate the distinction between the processes of induction and deduction, until we have first examined the characteristic features which are common to the two, and which constitute the essential elements of inference itself. The nature of inference may be unfolded in two ways. We may consider what it is in its outward aspect — that is, through its phenomenal manifestation in what it effects; or it may be more strictly defined in terms of its warrant, or ground. From the first point of view we examine inference as regards its psychological significance; that is, what is inference considered as a psychical experience, its nature and characteristics? But we must consider also the second question, whether there is any

necessity limiting and determining the subjective experience, which presents the character of a law having universal validity. What goes on in the mind during the process of inference? Also, what is the rationale of such a process? These questions we will examine more closely, in order to show the nature of inference under the two aspects, the one psychological, and the other logical.

It is a well-recognized fact in psychology, that in our simplest as well as the more complex perceptions, the interpretation of the data of presentation always goes beyond the strict content of the data themselves. We see more than is given in the field of vision immediately before us. The mind supplies here and there the necessary parts that are lacking in the actual elements of presentation, and yet which are necessitated by the known nature of that which is actually given. We form our judgment of distance indirectly, and not through direct presentation. So, also, our idea of a third dimension is acquired by a process, marvellously complex, in which the data both indicate and yet are transcended by the results. Whether the nativist or empiricist holds the true position concerning original psychical experience, it still must be conceded according to either theory that the development of our perceptions corresponds to a law of growth based upon accumulated inferences. Inference has been defined as the indirect reference of a content to reality, and as such, we see the beginnings of inference in the most simple of our perceptions. Every perception contains a direct

reference to reality, but also something which in a greater or less degree is referred indirectly to reality. The fact that our knowledge as given in the complete perception contains more than is actually mediated through the avenues of the senses, is due to the apperceptive processes of consciousness. Mind is active in perception, and not a mere passive receptacle. That which is given, the raw material of the senses, is elaborated and extended, as it is combined with the wealth of representative and conceptual material which the mind brings to every new perception. To this extent, at least, the mind possesses a creative function. A certain appearance of sky, combined with peculiar conditions of mind and temperature, leads one to assert, with some degree of certitude, that it will rain before morning. The prediction is an inference based upon, and growing out of, the actual data of perception, and yet far outrunning them. We recognize a friend from his step or voice. The mere presentation is only a sound. That it is associated with a person, and not an animal, or a thing, is an inference; that this is the particular person whom we recognize as a friend and can call by name, even before we turn around to confirm the opinion by direct testimony of vision, this is a still further inference. And even when we open our eyes in simple vision itself, we fill up many a gap in our minds, and give depth and distance, and interpret the contrasts of light and shade, and the play of colors, through the process of inference, although we may not be aware of the process itself,

which is automatically operative through long-continued habit. When we thus regard inference as a psychological phenomenon, it may be readily explained by the laws of comparison, association, recognition, generalization, etc. And, as such, inference has a subjective force at least, and leads to the habit of prediction and expectation. The will, influenced by the resulting belief, leads to activities consistent with such expectation.

Here, however, the question arises, which is urged with such force by Hume, Is there objective validity as well as subjective necessity? This leads to a consideration of inference, from the second point of view, above mentioned. We may be constrained to believe certain things concerning the great world lying beyond the sphere of immediate consciousness; but what warrant have we in so doing, or what assurance that our conclusions are correct? May we not be deceived, after all, and by some psychological trick be led to regard the phenomena of consciousness as quite otherwise than that which obtains in reality? We may have a strong aversion to sitting down at a table where the number of persons will be thirteen. But has the subjective conviction, that one of the thirteen will die in the course of the year, any value when we come to refer it to reality, and ask ourselves the nature of the ground upon which the conviction is based?

On the other hand, however, it is quite a different kind of necessity which constrains us to judge that if a person jumps off of the roof of a house,

he must surely fall to the ground below. Some grossly superstitious and ignorant people may believe the former with as obstinate a conviction as the latter, so that a purely psychological criterion of the strength of conviction is not at all adequate or satisfactory. Is there any other criterion? In what instances does this subjective constraint proceed from the necessities of reality? or, in other words, in what cases are we able to discover a logically grounded warrant which compels the inference, in distinction from the mere psychological compulsion which is occasioned by the psychical tendencies of association and generalization?

This leads us to consider the logical, in distinction from the psychological, nature of inference. Inasmuch as the characteristic feature of inference consists in this, that while depending upon certain data of presentation, it nevertheless wholly transcends them, the question naturally suggests itself, whether it is something within the data themselves, or without, by virtue of which the mind thus goes beyond them in the process of inference. If it lies wholly without the data, it must be something imposed upon them by the mind, and as such can have only a psychological force and value. For instance, the belief that if thirteen sit down together at a table, one will die in the course of the year, can have only a subjective value and significance. This is true in all cases where the necessity of conviction finds its origin in prejudice or in superstition, or it may be in the force of authority. In all such instances we feel the lack of a satisfactory logical ground. How-

ever, on the other hand, if the data of consciousness contain within themselves that which enables us to transcend them at the same time that we interpret them, there is external validity for our inference that has a logical worth. This seems at the first glance to be a paradox. How can any content enable us to state concerning it more than is contained within it? The answer to the seeming paradox is that every concept, and every perception as well, have both an explicit and implicit content. We never attain complete vision or perfect apprehension.

There are, moreover, many points of view, each giving additional knowledge concerning any phenomenon present in consciousness. We see, therefore, only in part, and yet that which is seen contains certain necessary implications concerning that which is not seen. In the progress of knowledge, subsequent observations, different points of view, are ever confirming and amplifying our inferences, enabling us to perceive immediately what formerly was only inferred. The process by which the implicit is becoming explicit indicates a necessary relation existing between that which is known mediately and that which is known immediately. Moreover, consciousness has been represented as a stream, or an intricately interwoven web, — something extremely complex. Every part is related both proximately and remotely. There is no such thing as an isolated perception; every perception has its complex relations and connections. So also every concept which is formed by generalization through comparison and abstraction, of our presentations as interpreted by

us, possesses this characteristic of greater or less complexity. In this manner the world of consciousness is constructed; that is, the world as it is for us. This forms a complex whole made up of parts, which in themselves may be regarded as wholes, and yet which may be still further divided and subdivided.

Such an interrelated whole we may style a system, or, in other words, a complex whole whose parts are congruently arranged. The idea of system finds expression in the "Law of Totality," — that our knowledge is capable of arrangement in a self-consistent and harmonious system, and which moreover in its content and form faithfully represents objective reality.[1] We find, therefore, that in the focus of consciousness at any one time, whether in the sphere of presentation or in the region of representative or the conceptual processes, whatever is given carries with it always certain implications, and therefore certain necessary relations. This is specially emphasized in Bosanquet's definition of system: "System is a group of relations, or properties, or things, so held together by a common nature that you can judge from some of them what the others must be."[2] Two facts regarded as independent and considered separately may give no information beyond their explicit contents; but when conjoined, they imply more than the sum of their parts. How often two ideas in separate minds yield no result; but brought together, they give light. Isolation

[1] Ueberweg, *A System of Logic and History of Logical Doctrine*, pp. 540 f.

[2] Bosanquet, *The Essentials of Logic*, p. 140.

negatives inference. To unfold any data in all their manifold implications is the process of inference. Its warrant lies in the fundamental postulate of knowledge which we are constrained to assume; namely, that our consciousness must be self-consistent throughout. Whatever is admitted as true must find a congruent place in the system to which it is possible to refer it. The necessity of fitting it in its proper place gives rise to certain implications which necessitate corresponding relations and attributes. And if it could not be put into such a place, we would feel that we should have to surrender the idea of self-consistency in the variously related elements of our consciousness. The very integrity of our mental life necessitates this conviction.

Therefore a part being given, we supply in our minds other parts, or the whole to which the given part must necessarily belong. To achieve this, with logical warrant, our knowledge of the part must be adequate to the extent that we know that the element under consideration cannot be complete in itself, but must be supplemented by its appropriately related elements which with it go to make up the complete system. We infer the nature of the flower not yet in bud, by the sprouting leaf. The one necessitates the other by virtue of their common inherence in the same plant system. We know that figs do not come from thorns, nor grapes from thistles. Columbus, noting the seaweed, and birds, and the drift of the sea, inferred a shore beyond, to which he was constrained by the necessities of thought to refer them. It is said of Cuvier

that he was able to reconstruct part for part the entire frame and organism of an animal whose fossil tooth alone formed the original datum. He knew the system to which it must have belonged and to which it alone could possibly be referred. An interesting quotation from Cuvier himself illustrates most appropriately this function of inference. He says in his *Ossemens Fossiles*, "I doubt if any one would have divined, if untaught by observation, that all ruminants have the foot cleft, and that they alone have it. I doubt if any one would have divined that there are frontal horns only in this class; that those among them which have sharp canines for the most part lack horns. However, since these relations are constant, they must have some sufficient cause; but since we are ignorant of it we must make good the defect of the theory by means of observation: it enables us to establish empirical laws which become almost as certain as rational laws when they rest on sufficiently repeated observations; so that now whoso sees merely the print of a cleft foot may conclude that the animal which left this impression ruminated, and this conclusion is as certain as any other in physics or morals. This footprint alone, then, yields to him who observes it the form of the teeth, the form of the jaws, the form of the vertebræ, the form of all the bones of the legs, of the thighs, of the shoulders, and of the pelvis of the animal which has passed by."[1]

In the common conduct of every-day life we infer

[1] Quoted by Jevons, *Principles of Science*, 2d ed. p. 683.

beyond the immediate present experience to future happenings and in a similar manner. My train is half an hour late. I know I must miss my connections at the station ahead; for the train I am hoping to catch at that place is scheduled to leave five minutes after the time of arrival of the train I am now on. The time relations here necessitate my missing my connections. This is rendered still more certain if they are rival roads; on no account will one wait for the other. Moreover, the train I hope to make is made up and leaves the station in question, and so I cannot fall back upon the favoring chance that it also may be detained en route, and so enable me, after all, to reach it in time. Thus, with every additional knowledge of the system which forms the ground of my inference, and the various conditions which affect it, the validity of my inference is thereby increased. Inference regarded as the analysis of a system of interrelated parts is illustrated in the following paragraph of Professor James: "The results of reasoning may be hit upon by accident. Cats have been known to open doors by pulling latches, etc. But no cat, if the latch got out of order, could open the door again, unless some new accident at random fumbling taught her to associate some new total movement with the total phenomenon of the closed door. A reasoning man, however, would open the door by first analyzing the hindrance. He would ascertain what particular feature of the door was wrong. The lever, *e.g.*, does not raise the latch sufficiently from its slot — case of insufficient elevation — raise door bodily on

hinges! Or door sticks at top by friction against lintel — press it bodily down! I have a student's lamp of which the flame vibrates most unpleasantly unless the collar which bears the chimney be raised about a sixteenth of an inch. I learned the remedy after much torment, by accident, and now always keep the collar up with a small wedge. But my procedure is a mere association of two totals, diseased object and remedy. One learned in pneumatics could have named the *cause* of the disease and thence inferred the remedy immediately."[1]

Inference, therefore, may be regarded as a deep penetrating insight. The explicit is that which lies upon the surface, which the mind immediately grasps, for it lies directly in the focus of consciousness. Whereas the implicit is beneath the surface, and is revealed only through a searching analysis. This difference may be exhibited through the distinction between the actual and the potential. A child regards gunpowder merely as a pile of coarse-grained sand. The man sees what the child sees, but also the existing possibilities under certain conditions of explosive force. He apprehends the potential as well as the actual; and his inference as to the possible results is based upon his superior insight. It is therefore the well-furnished mind which sees things as most widely related, and discerns the potential as well as the actual manifestation, which will prove the most fertile in accurate inference, in prophetic suggestion, and in inventive resource.

[1] James, *Psychology*, Vol. II. pp. 339, 340.

The whole world of reality, as well as that of knowledge, may be considered as one system, embracing within the unity of its totality all the various systems with their complicated parts. From this point of view everything bears relations to everything else in the universe. The original signification of the term universe is thus emphasized. This thought, no doubt, Tennyson had in mind in the following verse: —

> Flower in the crannied wall,
> I pluck you out of the crannies,
> I hold you here, root and all, in my hand,
> Little flower — but *if* I could understand
> What you are, root and all, and all in all,
> I should know what God and man is.

We can, in this connection, best exhibit the precise nature and function of the universal in inference. The possibility of unfolding the properties or relations of anything in all its implications depends upon our knowledge of the universal concept to which the properties or relations in question are naturally referred. While a singular proposition is the statement of the mere occurrence of a phenomenon, the universal always implies a knowledge of the conditions and relations of the phenomenon.[1] Insight is only possible where there is a wealth of universal concepts. We see an animal which we observe to be cloven-footed. We infer that it also chews its cud. We do not observe this. The assertion does not arise

[1] See Green, *Philosophical Works*, Vol. II. pp. 284, 285.

directly from observed reality, but indirectly through the generic concept that has grasped together the two attributes of chewing the cud and cloven feet as always and necessarily coexisting in one and the same animal Inference, in this sense, may be regarded as the indirect reference of knowledge to reality, and this is always mediated through the universal. The universal has this characteristic feature, that it preserves an identity in the midst of manifold differences. The same thought may be expressed by saying that the universal manifests a unity in the midst of diversity. However widely different, in many respects, the animals may appear that chew the cud, — as the cow, deer, sheep, etc., — there is always the constant characteristic that they are cloven-footed.

Such a point of identity furnishes the constant factor which determines the nature and the validity of the inference. Were it not for this conceptual power of the mind, this ability to grasp phenomena in their universal essence, and consider them as interrelated and connected, we could never pass beyond individual and particular experiences which would form a series of wholly disconnected events. Knowledge could not then form a self-consistent system, or inference possess any higher worth than a haphazard guess. As Green says, "A 'mere fact,' a fact apart from relations which are not sensible, would be no fact, would have no nature, would not admit of anything being known or said about it."[1]

[1] Green, *Philosophical Works*, Vol. II. p. 301.

Moreover, inference is not merely employed to extend the field of consciousness in unfolding supplementary elements lying beyond the sphere of direct cognition; the elements may all be given immediately, and inference employed to discover their connection and interrelations, by virtue of what bond they belong in one or the same system. Inference here functions as explanation. A man is found dead; there are many wounds upon his person, evidences of a struggle in an out-of-the-way place upon a lonely road. Such a combination of facts calls for an explanation which shall be consistent with them. The facts must all be correlated in a system whose related facts and the unity of the whole will completely satisfy the mind. The mind is satisfied only when all hang together in what seems the only possible self-consistent co-ordinated system. The facts being given, they must be read backward to their origin. The other aspect of inference is the reading of facts forwards, or unfolding them in their necessary consequences. Inference is the reply to the natural questions of the mind, — whence and whither? And the process is essentially the same, whether its peculiar mode consists in the evolution or the involution of that which is given in consciousness.

Moreover, the mere psychological inference, the subjective extension of the data of consciousness without any objective ground or warrant, should ever be corrected, or even at times wholly set aside by means of the truly logical inference. Where

the psychological experience, in transcending simple presentation, proceeds upon strictly logical grounds, and has objective validity as well as subjective necessity, we possess a warrant of the highest possible worth.

## CHAPTER II

### Induction and Deduction

There have been divergent tendencies in the history of logic, to make either deduction or induction alone the whole of logical procedure in the process of inference. The fact that the Aristotelian logic, which is essentially deductive, has been for centuries exclusively associated with logic as a whole, has left the impression upon many minds that it is the beginning and end of the logical encyclopædia. On the other hand, J. S. Mill and his followers have attempted to analyze the syllogism to prove its essentially inductive character; and they have maintained that all reasoning is inductive. This is the position in the main of Bacon, Locke, and Bain. Locke, for instance, insists that the syllogism is of less value than external and internal experience, induction, and common sense.[1]

So also, in a similar vein, Schleiermacher says: "The syllogistic procedure is of no value for the real construction of judgments, for the substituted judgments can only be higher and lower; nothing is expressed in the conclusion but the relation of two terms to each other, which have a common

[1] *Essay on Human Understanding*, Book IV. p. 7.

member, and are not without, but within, each other. Advance in thinking, a new cognition, cannot originate by the syllogism; it is merely the reflection upon the way in which we have attained, or could attain, to a judgment, the conclusion; no new insight is ever reached."[1] The two opposed views thus indicated do not necessitate conflicting or mutually exclusive processes. It is better to regard them, not as radically different types of inference, but rather as different phases of one and the same inferential process. We have seen that inference consists in interpreting the implications of the system to which the given in consciousness belongs. In the light of this definition we can best indicate the relative functions of induction and deduction in the process of inference. When the system can be considered as a whole, and is apprehended in its entirety, then it may become the ground upon which the inference is based, resulting in unfolding the necessary nature or relations of any of the parts considered in themselves, or in reference to the system as a whole. The procedure in such a case is from the nature of the whole system, to the nature of the several parts, and their existent relations, and this is deductive in its essential features.

On the other hand, when we know the various parts, and proceed from them as data to construct the system which their known nature and relations necessitate, it is induction, or procedure from elementary parts to the whole thus necessitated.

[1] See Ueberweg, *System of Logic*, etc., p. 345.

From a knowledge of the planetary system, we can infer the necessary positions of sun, moon, and earth at any required time, as, for instance, in the calculation of an eclipse. This is deduction. But when we begin with investigating the several movements of the different planets, and from them infer the necessary nature of the system of which they are parts, we have the process of induction. Such processes we see must be complementary, and mutually dependent. As Lavater says, "He only sees well who sees the whole in the parts, and the parts in the whole."

Moreover, the distinction between deduction and induction may be shown through their respective relations to the universal, which we have seen is the ground of inference. The question whose answer leads to the deductive process in reasoning, is, What does the universal necessitate? In induction, the question which starts the investigation is, Into what system may I construct the given material properties or relations, so as to reach a universal concept that will be consistent with itself and with the whole of knowledge which forms the world of consciousness? In this there is an analytical discrimination of the essential and accidental elements, and the gathering together of the former into the complex whole which is the universal. Induction, therefore, is inference viewed from the side of the differences; deduction is inference viewed from that of the universal. For instance, we may investigate the characteristic features of a diamond, and find that a certain specific gravity, 3.53 as compared with

water, is a constant and determining attribute, and as such must be incorporated as an essential element of the general concept diamond. We can then form the universal judgment. Whatever stones possess this specific gravity are diamonds. Their differences, regarding size, brilliancy, etc., may all be set aside as accidental, but the one constant determining feature indicates a oneness in which they all agree.

And so with the other essential attributes. After possessing such knowledge gained inductively, we may use it practically in a deductive manner; and it is so used in discriminating between true and imitation stones, as described in the following process: "Diamonds, rubies, and sapphires are now tested by floating to prove their genuineness. The liquid used has five times the density of water, and is composed of double nitrate of silver and thallium. The tests are rapidly made, as all stones of the same nature have the same specific gravity, while none of the bogus ones have the same weight as those they are made to imitate."

Another view of the relation of induction to deduction may be gained by calling attention to the difference of significance between the terms, a truth and a fact. A fact carries with it only the special and individual character of the particular occurrence in which it is manifested. A truth, however, is always universal in its very nature, admitting of universal application, and capable of illustration in an indefinite number of different facts which embody its essence. In deduction we have given some truth of universal nature that leads to indi-

vidual facts that may be subsumed under it. In induction, we interpret a fact or a number of facts in the light of their universal implication, on the ground that there can be no such thing as an isolated fact, but every fact must have some relation to a universal to which it must be referred.

While considering the distinctions between induction and deduction, we must not overlook their mutual dependence. We cannot proceed in deduction irrespective of induction, because the universal upon which the deductive process is based arises in the majority of cases from a previous induction. It is true that the universal term may be in a proposition that is known *a priori*, as the axioms of geometry and certain space and time postulates; but a very small proportion of major premises can be said to have such an origin, and their resulting conclusions have very slight material significance. Deduction that reaches other than purely abstract and formal conclusions must rest upon induction for the material to form its premises. We find this even in the technical construction of the syllogism, where, for instance, the question of the distribution of the terms is raised. We may insist that a certain middle term is distributed as it is the subject of an universal affirmative proposition; but then the further question naturally suggests itself, How do we know that the proposition in question is really a universal? Its material significance alone tells us that we may write it as an *A* or *I* proposition, as the case may be. The matter is a function of the form,

and the form a function of the matter. They cannot be separated in fact, unless we conceive reasoning as a purely formal process of determining a conclusion, irrespective of the truth or falsity of the premises. If we regard the premises as given, and we accept them with unquestioning credence, the deduction is purely formal; so also if the various terms are expressed by letters *A*, *B*, *C*, etc., and devoid of any material significance. Any process of reasoning based upon a slavish acceptance of premises can only reach artificial and even false results. In the actual experiences of life our premises are not made for us. They must be constructed by us through our interpretation of reality. Disregard of this has brought formal logic into much disrepute, and it has often degenerated into the barren discussion of logical puzzles and quibbles. Grant a person any premises he may choose to assume, irrespective of an inductive test of their validity, he can prove black white, and white black.

On the other hand, induction is dependent upon deduction; for we cannot reason from particular instances to a universal proposition, unless we assume as basis of the whole inductive process some postulate which has real universal significance. Otherwise, we reach only a high degree of probability, but not necessity; a rude generalization, but not universality. When we assert some such general statement as this, that arsenic always acts as a poison, we have the universal character of the proposition upon an underlying postulate that is

understood even though it is not expressed, such as the uniformity of nature, that under identical conditions we always look for identical effects. This will be discussed later more in detail; it is referred to at this point merely to illustrate the deductive basis of induction. Bradley insists that there can be no such thing as induction, because it always rests upon an implied universal which gives to the process as a whole a deductive character.[1] His criticism has the force only of proving that induction cannot be independent of deduction. This dependence does not, however, necessarily vitiate the integrity of induction as a mode of the inferential process. Lotze has placed special emphasis upon this dependence of induction upon deduction. He says: "It is the custom in our day to collect into one body the numerous operations which assist us in ascending from particulars to generals, or to call this inductive logic, and to set it against the deductive or demonstrative logic along with much disparagement of the latter. Such disparagement rests on a mistake. The inductive methods, it is certain, are the most effectual helps to the attainment of new truth, but it is no less certain that they rest entirely on the results of deductive logic." [2]

Moreover, in induction the results obtained and formulated in general propositions may be extended, and often modified by a deduction which is based

[1] Bradley, *Principles of Logic*, p. 332.

[2] Lotze, *Logic*, p. 288. See also Bosanquet, *Logic*, Vol. II. p. 119.

upon them as major premises; for the deduction thus proceeding from them reveals new instances which conform or perhaps modify the simple inductive results themselves. What is popularly called a hasty generalization, if made a major premise of a syllogism, will often lead us astray through the deductions drawn from it. As soon as we are aware of this, we return to question the validity of the generalization, whose weakness is not appreciated until thus tested and revealed. Thus deduction serves to extend and correct the results of induction, and at the same time it itself is dependent upon the results of inductive generalization for the material to form its premises. We come to see, therefore, how intimately associated these two processes are in actual reasoning. For convenience of illustrating their individual characteristics, they may be considered as separate, and each investigated as an independent mode of inference. But they are in reality mutually related and dependent, and are always found manifesting their functions together. In any course of reasoning concerning the conduct of our every-day affairs, or in scientific investigation, anywhere, indeed, outside of the artificial examples of logical text-books, we reason both inductively and deductively in one complex process.

## CHAPTER III

### The Essentials of Induction

We now proceed to a more precise determination of the nature of induction. Its point of view in all reasoning regards concrete instances. They are the data, and from them general propositions are to result. The procedure is from given facts to laws which are the ground and explanation of these facts. We are here, however, at once struck with the evident break in the course of our reasoning. Procedure from the particular to the universal cannot be a continuous process. There is a gap somewhere. The conclusion contains more than the premises. In deduction, we are proceeding from the greater to the less, and we experience no violation of our logical sense; but at once we appreciate the difficulty which attends the reverse process, from the less to the greater. Here we soon reach a point where we pass beyond the sphere of our experience to the generalization which necessarily embraces far more than our experience. This is the so-called inductive leap; or it is sometimes referred to as the inductive hazard. But is this a leap in the dark — a wild guess concerning all that lies beyond the sensuous sphere of our immediate experience? This

would be the case, were we compelled to use the mere data of experience as sole ground for our inferences. John Stuart Mill insists that nothing whatever is given in consciousness but particular sensations, and these are but subjective states of feeling, and with no assurance of any definite correspondence with the external world. With such purely empirical data it is impossible to proceed to truths of universal validity. It is necessary to postulate some universal truth which the mind through strictly *a priori* considerations is constrained to formulate, and which will serve to bridge the gulf between the particular and the universal.

This postulate has been variously expressed by different authors, yet with substantially the same significance in all. In the older logic, it is put under the convenient formula of the uniformity of nature; that is, that beyond the sphere of experience, phenomena will behave in the same manner, under like conditions, as in the sphere of immediate observation and experiment. In the modern logic this is somewhat differently expressed. The phrase "uniformity of nature," being somewhat indefinite and implying a point of view purely objective, is not used. Modern writers have omitted it largely from their terminology. Lotze says: "The logical idea upon which induction rests is by no means merely probable, but certain and irrefragable. It consists in the conviction, based upon the principle of identity, that every determinate phenomenon $M$ can depend upon only one determinate condition, and accordingly that, where under apparently dif-

ferent circumstances or in different subjects *P*, *S*, *T*, *U*, the same *M* occurs, there must inevitably be in them some common element Σ which is the true identical condition of *M*, or the true subject of *M*."[1] We have a somewhat similar description of the basis of the inductive process given by Sigwart: "The logical justification of the inductive process rests upon the fact that it is an inevitable postulate of our effort after knowledge that the given is necessary, and can be known as proceeding from its grounds according to universal laws."[2] Bosanquet considers as the basis of inductive inference that which he calls the postulate of knowledge, that "the universe is a rational system, taking rational to mean not only of such a nature that it can be known by intelligence, but further of such a nature that it can be known and handled by our intelligence."[3]

I have quoted these passages from Lotze, Bosanquet, and Sigwart, that we may appreciate the modern tendency to derive the inductive postulate from an epistemological source; namely, that our knowledge must be consistent throughout with itself, part to part, and parts to whole, and that the world for us is the world as constructed by our knowledge. Whatever is given in consciousness must belong therefore in the one place where it appropriately and necessarily belongs. Here also there must be a place for everything, and everything in its place. There must be a uniformity of consciousness; that

[1] Lotze, *Logic*, p. 102.
[2] Sigwart, *Logic* (Eng. translation), Vol. II. p. 289.
[3] Bosanquet, *The Essentials of Logic*, p. 166.

is, the primary postulate and the uniformity of nature is secondary to this, and implied in it. This postulate may also be expressed as follows: What is once true, is always true. Here true is used in the sense of the universal significance of a fact. Whenever a concrete instance is present in consciousness, its existence must be considered as necessitated by some antecedent which can satisfactorily account for it, and which can at the same time be appropriately adjusted to the whole of our knowledge in interpreting it. Bosanquet says that "ideally speaking every concrete real totality can be analyzed into a complex of necessary relations."[1] These necessary relations of course have a universal significance, and therefore in every concrete instance, if we can rightly interpret it, we may discern the universal element which is contained in it, and gives it a place and meaning in the world as cognized by us.

There is a sense in which induction may be regarded as the inverse process of deduction. In deduction the problem is concerned with the question, What does the universal necessitate? In induction, the instance is given, and the problem is, What universal can be discovered which could give rise to the instance in question? This view of induction is especially associated with the name of Jevons, whose inductive system is described as the inverse of deduction. He calls it the deciphering of the hidden meaning of natural phenomena.[2]

[1] Bosanquet, *Logic*, Vol. II. p. 82.
[2] Jevons, *Principles of Science*, p. 124.

The name commonly used to designate this view of induction is that of "reduction," originally suggested by Duhamel.[1] This process was known to the old logicians, who called it "Method" to denote the process of hunting for middle terms by the aid of which a given conclusion could be proved.[2] Like all inverse processes, it is by itself an indeterminate one.

Given All $A$ is $B$, and
All $B$ is $C$,

we infer by the direct process of deduction that

All $A$ is $C$.

But in the indirect or inverse process we have given all $A$ is $C$, and the problem, to find a middle term which necessitates such a conclusion, is an indeterminate one. There may be a number of middle terms. This is analogous to the method of integral calculus; while differentiation leads to a definite result, the inverse process of integration leads to an indeterminate result. So also we multiply two numbers, producing one determinate result; but inversely, when we have given a certain number, and ask what factors multiplied together could produce this number, we may reach several different solutions. The answer is indeterminate. Professor Jevons, in his scheme of inductive inference, falls back upon probability to indicate which of several possibilities is the most likely one in the

[1] Duhamel, *Methodes*, Vol. I. p. 24.
[2] Venn, *Empirical Logic*, p. 361.

given case.[1] But before the inverse operation can result in determinate results, the given terms such as $A$ and $C$ must be subjected to some analysis in order that their material signification may give suggestion as to the nature of the middle term. For instance, a man is found dead, washed ashore by the tide; the natural supposition would be that he met his death by drowning. And yet it might possibly happen that the man died through injuries inflicted by blows, or by poison, or heart failure. The attendant circumstances and bodily indications must suggest the most probable cause to account for the given effect. Venn criticises Jevons' view of induction, making it the inverse process of deduction, on the ground that it is purely a formal process, and therefore can lead only to indeterminate results.[2]

It is always possible, however, to make some analysis of the material significance of the data, as has been above indicated, which relieves the purely formal processes from the indefiniteness of the results. Bosanquet criticises Jevons' theory of inductive inference, in that the hypothesis proposed to account for the given in reality can at best be only highly probable.[3] However, Venn, Lotze, Bosanquet, Sigwart, all allow a place to the inverse function of all inductive reasoning; their contention, however, is this, that it does not furnish an adequate account of the whole matter.[4]

[1] Jevons, *Principles of Science*, p. 219.

[2] *Empirical Logic*, p. 359.

[3] Bosanquet, *Logic*, Vol. II. p. 175.

[4] Venn, 361; Bosanquet, Vol. II. p. 175; Sigwart, Vol. II. p. 203, 289. Lotze, *Outlines of Logic*, p. 93.

It is interesting to note that Whewell's theory of induction corresponds in the main to this idea of reduction, or inverse process. He finds in induction a twofold operation of the mind, consisting in the colligation of facts and the explication of conceptions. By the colligation of facts he refers to that insight which is able to see the connections and relations which necessarily exist between the different phenomena present in consciousness; and by explication of conceptions he refers to the appropriate fitting in of these related facts to some conception of the mind which most readily accounts for them.[1] Such a process is merely the reading of given facts backward to their origin, or substantially an inverse process, where the procedure is from the given concrete to the explanation of the same in terms of the universal to which it can be most appropriately referred. So also Mill's account of procedure by hypothesis, as we shall see later on, presents characteristics similar to this process of reduction.

The end of induction is to discover a law having objective validity and universal application. There is a distinction which must be noticed and clearly kept in mind; namely, the distinction between a law and a rule. Induction seeks a law, and not a rule. A law expresses the essential and universal relations subsisting between given phenomena, eliminating entirely all accidental and local coloring. A law has objective validity, and preserves a constant nature. There can be only one law in reference

[1] Whewell, *Philosophy of the Inductive Sciences*, pp. 172, 202.

to one and the same connection of facts. A rule, however, is subjective, dealing with the individual's attitude to phenomena, rather than an explanation of the essential features of the phenomena themselves. It often is determined in the concrete by that which is external, local, and accidental. There may be many rules, varying with many minds and many climes. Fundamental and universal laws of political economy become maxims and rules in different communities. The laws of morality, universal and immutable, become rules of conduct in individual experience admitting of wide difference of opinion and diversity of application.[1] In the processes of induction, therefore, the law is the desideratum, and not the rule.

Law, however, is used rather loosely in our ordinary terminology. Law as used in jurisprudence has a meaning quite different from law as used in physical science. And so, also, the laws of biology, the laws of political economy, the laws of ethics, are referred to with different shades of meaning in each sphere. However ambiguous may be the significance of "law" in ordinary thought and usage, nevertheless in induction it has a constant and a simple significance, which if carefully adhered to will avoid confusion, and obscurity as well, in our inferential processes and results. Law in induction is always in the form of an hypothetical universal:—

If $A$ is, $B$ is.

It does not assert what has happened, but what

[1] Lotze, *Logic*, p. 335.

should happen under certain conditions. Given the antecedent $A$, a certain determinate consequent $B$ is always necessitated. The relation is constant and invariable, and therefore has a universal significance.

Induction holds a peculiar and important place in our every-day life, because it has to do with the analytical treatment of instances as they appear in experience. The large part of our conscious thinking has to do with the concrete, the raw material of experience; this, induction alone can handle. Leonardo da Vinci's maxim was "to begin with experience and by means of it to direct the reason."[1] Thus the superstructure of knowledge is raised day by day. The given is continually being interpreted and referred to its appropriate place, as the stones of the quarry are hewn and fitted in their proper position in the building for which they have been designed. There are certain individual experiences which it is impossible to determine through our syllogistic forms. They cannot be judged deductively. There is no general category under which they can be subsumed. They may be formally illogical if thus expressed, and yet admit of direct investigation and experiment in an inductive manner, for the purpose of disclosing the law underlying them and as yet unknown.

It often happens that through indifference or indolence, we are content to refer many phenomena to long-established and convenient categories, which, if investigated independently, we would find could

[1] Ueberweg, *Logic*, p. 42.

not possibly be so treated. The convenient pigeon hole, because near at hand, receives much that does not properly belong there. It is the office of induction to investigate anew the old material, and then to reclassify in accordance with the revised generalizations which such investigations may necessitate.

The procedure by induction is in keeping with the scientific spirit of the day,—to interpret the phenomena of nature as given, and not to anticipate nature through preconceptions, and wrest fact in order to fit theory. It comes to the sources in nature with empty vessels to draw and carry away that which nature alone can give.

## CHAPTER IV

### Types of Inductive Inference

The process of induction, as we have seen, is a procedure from given instances to the discovery of the law which underlies them, and which is the ground of the connection of the various attributes and relations that unite in the one concrete whole. Viewed from the standpoint of the direction of the process, we have found that it is always towards some general expression of individual experiences, and in this respect it is the inverse of deduction, which proceeds from the general to the particular which is embraced in it. There is, however, another and important point of view that should not be overlooked. We have to consider the mode of the process as well as its direction; not merely the result to be attained, but also the peculiar manner of realizing the same must be considered. Difference in method here gives rise to various kinds of inductive inference. The end proposed in all is to generalize our experiences as they occur in the concrete and particular. When I find a given phenomenon, *A*, given in consciousness, and characterized by several distinctive features among which I note specially the mark *B*, the question at once most

naturally suggests itself Is there a reasonable expectation that I shall always find *B* as an inseparable accompaniment of *A*, so that I can assert confidently that whenever *A* is found, *B* also will be found? There are three ways of satisfying ourselves as to the existence of any constant rather than coincidental connection between antecedent and consequent, as *A* and *B*. These give rise to three different methods of inductive research, and they are as follows: —

I. The Method of Enumeration.

II. The Method of Comparison, or Analogy.

III. The Method of Scientific Analysis, or Search after Causal Connection.

Failure to distinguish between the three methods has given rise to confusion in the definition of and corresponding reference to inductive inference; some authors use induction in one, and some in another of these senses. It is necessary to discriminate carefully, and to maintain a strict consistency in the usage of the terms as defined.

I. *The Method of Enumeration.* — We observe the various instances in which certain attributes, as *A* and *B*, are conjoined in our experience. We count them in the sense of noting to what extent they accumulate, without noticing any exception to what seems at least an invariable connection. We do not necessarily count by precise enumeration reaching a numerically definite result. We notice merely to what extent the observed instances of like nature accumulate; that is, whether a few, a considerable number, or a very large number. The

mere number of instances produces a certain psychological impression, whatever may be their logical force. This is brought about through the laws of association, and creates an expectation of a continuous repetition of the experience in question. This arises from a natural tendency of the mind to generalize. We observe that crows are black; and the increasing number of confirming instances goes far to establish a connection between the crow and its color which seems to have universal validity. The enumeration of instances may lead us to any one of three results: —

1. We may meet with no exception whatsoever, until the scope of observation completely embraces the sum of all possible instances. This is complete enumeration, and when enumeration reaches this limit, it passes over into deductive reasoning, by virtue of the logical canon that whatever is true of the parts is true of the whole distributively; that is, provided the summation of the parts has been an exhaustive one. We assert that all the sheep of a given flock are white; for we have observed each separately and no one has been missed in the count. So, also, the judgment that all planets move around the sun, resulting from an enumeration of the planets one by one. It is possible also to have a perfect induction with an infinite enumeration of parts. This is possible in two cases, as pointed out by Beneke.[1] First, when the parts are connected together continuously in space, so that a survey of all is possible in a finite, and often a very short time. This occurs

[1] Quoted by Ueberweg, *Logic*, p. 482.

in geometrical demonstration when the inference, based upon the simple figure it refers to, is extended to all figures falling under the like definition. And second, when the parts are not continuously connected, if it can be proved syllogistically that what is true of a definite $n$th part, must also be true for the $(n+1)$th part.

Perfect induction also embraces arithmetical method and computation. Here the whole, which is the sum of the facts in each case, is a totality or universal whose differences, which are all separate and distinguishable, are yet homogeneous and equal.[1] There is no qualitative differentiation of parts, only a quantitative one. The total is the sum of the units, and each unit is like every other one. If we have one hundred units making a totality, the one that may be the twenty-seventh is precisely like the sixty-seventh. It is a case where each one counts for one, and no one for more than one in an absolutely literal sense.

It has been urged against perfect induction that it affords no new information, and, therefore, its results are not valuable. However, the summation of particulars in abbreviated forms is always an advantage. It is a labor-saving process to the mind. It enables the mind to retain a large number of facts by throwing them into one and the same category; and it facilitates arithmetical processes by convenient comprehending of units within a totality.

See Jevons' answer also

2. The second result that is possible, is that, in counting instances, our enumeration should prove

[1] Bosanquet, *Logic*, Vol. II. p. 54.

incomplete. From the necessities of the case, we are often not able to observe the entire sphere of possible occurrences and cover the whole ground. It may be that beyond the sphere of our experience, the constant connection between certain phenomena may be disturbed by the appearance of some variable factor of which we have been wholly ignorant. It is the possibilities beyond the sphere of observation which render uncertain the results of our count. We are sure as far as we have observed; but we have not gone far enough perhaps. Such results, formulated in general propositions, are termed empirical laws; that is, generalizations from an experience necessarily limited.

3. We have still a third case; where in our enumeration of positive instances we meet with exceptions to a greater or less extent. Here we cannot even sum up the actual experience in terms of a generalization. There are outstanding exceptions which will invalidate it. We must, therefore, fall back upon the theory of probability and the calculation of chances, presuming that, in general, we will meet with the same proportion of exceptions to positive instances in the future, that we have already observed in the past. So we make, in our minds at least, comparative tables of positive cases over against exceptions, and reach a summary of the result in the form of a ratio, whose numerator will be the number of positive cases observed, and the denominator the total number of instances including positive instances and the corresponding exceptions. We observe that

some cryptogamous plants possess a purely cellular structure; others, however, do not, being partially vascular. The probability that a new cryptogam will be cellular can be estimated only on the ground of the comparative number of known cryptogams which are cellular, as over against the total number of cryptogams, both cellular and vascular, previously observed.[1]

II. *The Method of Analogy.* — Here, also, we start with the experience that *A* is characterized by the mark *B*. But there is additional knowledge of which we may avail ourselves in the generalization of some past experience already effected, such as the following: that *A* very closely resembles *C*, in that the two have many properties or attributes in common. The inference by analogy is that *C* also, as well as *A*, will have the mark *B*. It may be that we cannot examine *C* in a number of various instances to see in how many the mark *B* occurs. Our only resource is the inference which is based upon the known resemblances, or analogies. This kind of inference, for example, was employed by Sir Isaac Newton in a very interesting manner. He had observed that certain "fat, sulphureous, unctious bodies," such as camphor, oils, spirit of turpentine, amber, etc., have refractive powers two or three times greater than might be anticipated from their densities. He noticed also the unusually high refractive index of diamond, and from this resemblance, based upon similarity in reference to one attribute only, he inferred that diamond also would

[1] Jevons, *Principles of Science*, pp. 146, 147.

prove to be combustible. His prediction in this regard was verified by the Florentine Academicians in 1694.[1] Brewster made a striking comment upon Newton's inference, to the effect that if Newton had drawn a like analogy in reference to greenockite and octahedrite as he did concerning diamond, inasmuch as they, too, have a very high refractive index, he would have been wholly incorrect. This is an indication of the fact that argument by analogy is not conclusive.

Bosanquet has very strikingly expressed the essence of the analogical method in saying that "in Analogy we weigh the instances rather than count them."[2] The distinction between analogy and enumeration of instances lies in this, that in the former we count similar attributes in the contents of two instances, and balance them against the dissimilar or unknown. In induction by enumeration we count similar instances, considering them in their totality without examination and comparison of their respective attributes.

III. *The Method of Scientific Analysis.* — The instance in question, *A*, which is characterized by the mark *B*, is subjected to a vigorous analytical examination, to show that *A* and *B* are related through a causal connection. This analysis is effected either through a minute observation or by means of exact experiment. The end to be attained by such analysis is to separate a complex phenomenon into its several elements, by which

[1] Jevons, *Principles of Science*, p. 527.
[2] Bosanquet, *The Essentials of Logic*, p. 155.

process a causal connection may be revealed, whose very existence is disguised by the complexity of the phenomenon. For instance, the phenomenon of death following the taking of arsenic is an event so complex as to evade a precise determination of the causal relation. When analyzed into simpler elements, it is found that the immediate effect of arsenic upon the bodily tissues is to harden them so as to prevent their normal functioning. This is the causal ground of the death due to arsenic. Moreover, this analytic process, which may be appropriately called a material one, is supplemented by a formal process of negation, that is, an instance in which the suspected causal element is absent in the complex phenomenon under investigation, and the related effect, before observed, now no longer appears. This formal process acts as a check, and as a verification as well, of the material analysis of the phenomenon. For example, an antidote, as sesquioxide of iron, being administered, no death from arsenic occurs; and it is also observed that no hardening of the tissues has resulted, therefore the former result, hardening of tissues producing death, has been thus corroborated negatively by the fact that where no hardening of tissues has resulted, death does not follow.

We see at once the advantage of such a method over that of counting all instances where taking of arsenic has caused death. The latter is a phenomenally adjudged result; the former penetrates with analytic insight to the ground of the phenomenon itself. Thus one instance, if its parts and their

manifold relations are adequately comprehended, may suffice for a universal conclusion based upon it. It is true, however, as remarked by Bosanquet, that "number of observations does, as a rule, assist analysis and contribute to eliminating error. Scientific analysis as such, however, does not deal with instances, but only with contents." [1]

In cases where the phenomenon does not reveal its component elements under observation, and it is impossible to subject it to experiment, the most likely cause of the effect in question is tentatively judged to be the real cause, until it can be verified in reality. This is procedure by hypothesis, and is always resorted to as preliminary to a subsequent experiment which is its test, or else in lieu of such an experiment when it is by the nature of the case precluded. It is a form of *ideal* analysis. The experiment is constructed mentally. The phenomenon is separated into what we would reasonably imagine its simpler elements would be. We are constrained to believe that if the hypothetical antecedent existed, it would be adequate to produce the effect. Although rising in the sphere of the imagination, it is that with which the mind is, for the time at least, satisfied as an explanation of the facts which demand some cause to account for them. Regarding induction as a process of reduction, hypothesis is the assumed universal, or middle term, which will necessitate the phenomenon under investigation as its logical conclusion.

We will now proceed to a further examination

[1] Bosanquet, *Logic*, Vol. II. p. 118.

of these methods, considered both singly and together.

1. They all proceed upon the supposition that what is given in consciousness has some necessary ground for its being. In enumerative induction, there is some causal connection presupposed, yet in a very general and indefinite manner, and accompanied by no analysis of the various concepts either by a systematic observation or experiment. It is a vague sense of uniformity which, when observed for many times, we feel will continue indefinitely. That which has happened often and not contradicted carries with it a certain convincing power by dint of bare repetition, especially to persons of narrow experience, and unaccustomed to discriminating observation. Ueberweg has made the following comment in reference to the so-called imperfect induction. "The conclusion is made universal with more or less probability, and the blank which remains over in the given relations of spheres is legitimately filled up partly on the universal presupposition of a causal-nexus in the objects of knowledge, partly on the particular presupposition that in the case presented such a causal-nexus exists as connects the subject and predicate of the conclusion. The degree of probability of the inductive inference depends in each case on the admissibility of this last presupposition, and the various inductive operations, the extension of the sphere of observation, the simplification of the observed conditions by successive exhaustion of the unessential, etc., all tend to secure its admissibility."[1]

[1] Ueberweg, *Logic*, pp. 483 f.

Analogy likewise proceeds upon the assumption of an underlying cause among the observed phenomena, and this is more definitely in the foreground throughout the process than in that of induction by enumeration. Analogy is based upon the postulate that similar phenomena have similar causes; the greater the agreement of the various attributes of the different phenomena compared, the greater will be the resultant probability that causes capable of producing them as effects will be similar. The similarity of the lightning flash to the electric spark suggested to Benjamin Franklin the possibility that they were due to a like origin, and by experiment his analogical reasoning was actually confirmed, as is well known. Upon the theory that the world as it exists for us in knowledge forms a system to some place in which every phenomenon given in experience must be appropriately and necessarily referred, it follows, therefore, that a simple experience, devoid of any complexity of parts, may fit into several possible places in our world of consciousness, and remain so far forth indeterminate. However, a complex phenomenon, with many parts intricately connected, will fit into one unique place only in the system to which it must be referred. It is like a key that will fit into only one lock. The presumption, therefore, is that any other phenomenon which resembles the first through much of its entire content, part for part, attribute for attribute, will also resemble it further as regards other attributes not yet examined, so as it will likewise fit into the peculiar place in the system of knowledge to which the

first has been found to belong. There is always a strong probability that agreement in spheres of great complexity is not a mere coincidence, but the result of a causal relation. One characteristic of a system, which we have found to be the ground of inference generally, is the co-ordination of like things under one concept. Analogy, therefore, is based upon the view of causal connections within the system which comprises the world as given in consciousness.

In the third method, the causal relation is more prominent still, and the search for it characterizes the procedure employed. That, which in the other methods may exist merely as a vague impression, is here formulated and made the direct and sole object of research.

2. The three methods in the order here presented show an increasing prominence given to the causal connection in the phenomena of experience. And therefore they possess a relatively increasing scientific value. As the first has only indirect reference to the causal connection of its facts, it is the least trustworthy and has no claim as a scientific method. It breaks down as soon as an exception is noted; and is even weakened by the fact that it is constantly menaced by the possibility at least of the appearance of an exception. "How do we know," says Green, "that the instances, with the examination of which we are always dispensing on the strength of the rule (that is, our generalization), might not be just what would invalidate it, if they were examined?"[1] We may arrive at the conclu-

[1] Green, *Phil. Works*, Vol. II. p. 282.

sion, based upon our observation and consequent record, that all sheep are white, and yet black sheep do occur, even in every flock, as the proverb has it. According to Aristotle, the proposition that all swans are white, was a perfectly general one, and yet in recent times black swans have been discovered in Australia. Bacon's criticism upon this method has become classic: "Inductio quae procedit per enumerationem simplicem, res puerilis est et precario concludit et periculo exponitur ab instantia contradictoria et plerumque secundum pauciora quam par est et exiis tantummodo quae presto sunt pronunciat."[1]

The validity of this method of procedure depends largely upon the probability of our meeting and noticing exceptions were they to occur. As Lotze puts it: "A man who never observes a place of public resort but once in every seven days, and that on a Sunday afternoon, has no right to suppose, because it is crowded then, that it is as crowded on a week-day."[2] He is here in no position to note the exceptions even should they occur.

Analogy, unless confirmed by experiment, or upon the ground of resemblance established by a verifiable hypothesis, has no claim to be considered as a scientific method. There may be false analogies depending upon surface resemblances. A child might conclude that oil would put out fire because it so closely resembles water, which he knows can extinguish the flames. The difference between essential and accidental agreement between phenomena

[1] *Novum Organon*, i. 105. [2] Lotze, *Logic*, p. 343.

can be revealed only when the underlying cause is ascertained.

The third method alone has scientific worth. True induction must be a continued search to discover a causal relation.

3. The two first processes fulfil their functions largely as tentative and suggestive methods. In enumeration of instances, we are often led to note resemblances which become the basis of analogy. And analogy suggests, in turn, hypothesis which is capable of verification through subsequent experiment.

The question may be put, "Which of the three processes is induction proper?" The fact is that it may involve all three, but it is not complete until it reaches the third, — the experimental method. Analogy is especially fertile in suggestion. Scientific minds most carefully trained and versed in scientific methods of research are often most keen in noting resemblances, and detecting analogies which become the basis of their experiments. Newton possessed that rare insight which, in spite of the manifest dissimilarity of the two phenomena, could yet discern an essential likeness between the fall of an apple and the gravitating force of the moon towards the earth.

4. It is also to be observed that the choice of method will depend largely upon mental habit. Some minds naturally or by special training and surroundings are given to experiment. They have a testing facility and inventive capacity. Others naturally are susceptible in an unusual degree to

contrasts and resemblances. Others again are accustomed to accurate observation that is ever pushing beyond and seeking to extend its sphere. Thus we have a natural division of these methods according to psychical proclivities. The choice of method is often conditioned by the force of circumstances. Experiment is not alway possible. Are all crows black ? There is no connection between the general organism of the crow and its color that has thus far been revealed through analysis or experiment. The only recourse is to number instances over the widest possible field. We say, moreover, that Mars may be inhabited; for it has an atmosphere similar to the earth and therefore capable of sustaining life. Analogy is the only guide in such a case, and it is impossible to verify it either by observation or experiment.

5. All the methods tend to one end, that of effecting a generalization of experience. The generalization may be either a numerically general one, or one expressed in terms of a generic concept.

(1) The former consists in the extension of several instances to their repetition under like conditions.

(2) The second consists in the extension of several instances to all cognate species under the same genus.

Examples of these two kinds of generalization are as follows : The general proposition that all sulphur is combustible is of the former kind; all instances are substantially of the same nature, and do not differ as distinguishable species under the same genus, but rather a repetition of like phenom-

ena. The general concept in the above proposition is of the nature of an *infima species.* On the other hand, the proposition that all mammals are vertebrates, has the subject-term in form of a generic concept. Many species, differing widely among themselves, may be embraced under it.[1]

[1] Sigwart, *Logic*, Vol. II. pp. 310, 311.

# CHAPTER V

## Causation

We have seen that induction as a truly scientific method consists in the analytical determination of the relations of cause to effect in any complex phenomenon, accompanied by a generalization of the result obtained. The final outcome of such a process is an universal concept which embodies a law, expressed in terms of a constant connection between antecedent and consequent. As Green has said, "The essence of induction consists in the discovery of the causes of phenomena."[1] A causal view of the universe gives rise to logical concepts, whereas a mythological view of the universe, as in ancient times, resulted in mere empirical concepts, which gave no assurance either of stability or invariability. It will be necessary, therefore, to determine more precisely the logical significance of the causal idea, which seems to underlie all inductive inference. This is no easy task. According to Clifford, cause has sixty-four meanings in Plato, and forty-eight in Aristotle.[2]

[1] Green, *Phil. Works*, Vol. II. p. 284.

[2] Clifford, *Lectures and Essays*, Vol. I. p. 149.

The causal idea has sometimes found expression in the phrase, the uniformity of nature, or it is often referred to as the doctrine of universal causation. These two phrases are often used interchangeably; this gives rise to confusion of thought, for their meanings are quite distinct. Uniformity of nature, strictly interpreted, means that like antecedents, under precisely the same conditions, will be followed by like effects; this idea expresses one phase of causation, viz. its invariability. The doctrine of universal causation, however, expresses the impossibility of phenomena rising spontaneously, without an antecedent, or antecedents, sufficient rationally to account for them. The two ideas lie at the root of the causal idea. As Tennyson has put it: —

> For nothing is that errs from Law.

Some confusion has also arisen from the failure to discriminate precisely between the philosophical and the purely logical questions relative to the general subject of causation. Causation may be viewed from three different points of view: —

1. What it is phenomenally, that is, as regards its physical aspects.

2. What it is essentially, as regards its real nature. This is a metaphysical question.

3. What it is in respect to its characteristic attribute of invariability. This is a purely logical question.

(1) As to the first, what is causation phenomenally? What is its purely physical significance? Investigations in this line have led to the doctrine

of the conservation of energy. This is substantially the assertion that, in every event, no new energy is called forth which did not exist before, potentially at least, nor can any energy be ultimately lost; nothing new is created, — there is only a change or transfer from one state or condition to another. Moreover, the sum total of energy in the universe is a constant quantity; it can neither be added to, nor subtracted from. There is an excellent illustration of this theory in the admirable chapter on "Conservation of Energy" by Professor Tait. I give it somewhat in full: "I allow an electric current to pass through a galvanic battery and there is for the moment a certain quantity of zinc consumed, or, as we may put it, a certain quantity of potential energy in the battery has been converted into the kinetic energy of a current of electricity. That current of electricity passes round some yards of copper wire, coiled round a bar of iron or a number of fine iron wires which are standing vertically inside this apparatus. The moment the current passes, these iron wires are converted into magnets, but, in consequence of the conservation of energy, while this is going on they weaken the current. The current of electricity becomes weaker in the act of making the magnet, but the moment the magnet springs into existence, it again is weakened, because, from the necessities of its position, its mere coming into existence necessitates the passage of a new current of electricity in another coil of wire which surrounds this externally, and finally this last current we can use to produce

heat, or light, or sound."[1] In this cycle of changes we see how closely connected even disparate phenomena are, and how the appearance of energy in any one definite state is dependent upon its previous existence in some other state. The doctrine of conservation of energy, we shall see later on, may be suggestive as to the nature of the analytical treatment of cause and effect.

(2) The philosophical question as to the inner nature of causation met with one answer generally until the time of Hume; namely, that the idea of cause signified that the antecedent was efficient in producing the corresponding consequent, implying the transfer of power sufficient to bring about the effect. Hume, however, contended that in the greatest possible extent of our knowledge, all that we certainly know is this, that one event follows another. We have no ground for an assertion concerning the manner in which the sequence is effected, nor assume any real tie between them. Hume insisted that phenomena were conjoined, but never connected.[2] His opponents, as Kant and others, deny him, however, his fundamental position, — that the origin of the causal concept comes from experience alone. They urged that it has an *a priori* origin, a concept simple and unanalyzable, given through intuitive insight; developed in the sphere of experience, but not dependent upon experience for its warrant. It is an interesting fact that the idea of the conservation of energy devel-

[1] Tait, *Recent Advances in Physical Science*, pp. 76, 77.

[2] Hume, *Essay on Idea of Necessary Causation*.

oped subsequent to Hume's time. It seems to give evidence which Hume insisted was not and could not be forthcoming, namely, concerning the idea of the antecedent as an efficient power. Through the modern doctrine, the impression of a transfer of real power is produced, though its mode and manner still remain a mystery.

(3) The logical aspect concerns not the phenomenal manifestation of cause and effect, nor their inner nature, but rather the element of invariability in causation. Two questions here suggest themselves: First, Is invariability a fact, — a constant element in causation? Second, How do we account for its existence? The first only has truly logical significance. The invariability of causation, that like antecedents under precisely the same conditions produce like effects, alone makes induction possible. Mill says that it is the belief in the uniformity of nature which stands as the ultimate major premise in every process of induction. Hume accepted it, and based inferences upon it, and never challenged it as a working basis as regards the affairs of every-day life. He acknowledged the element of invariability, and only denied the bond of connection. This element has peculiar logical significance: without it, it would be impossible to extend our knowledge beyond the seen and the heard, indeed that which is seen and heard would then have no meaning, and no basis for their interpretation and appreciation. Being assumed, however, in our logical postulate, we have a basis for induction, — a constant to be sought for, and to

be depended upon, in explanation of the past and in prediction of the future.

When we come to the second question, which is essentially a genetic one, how the belief in the uniformity of nature arose, we find two classes which answer respectively that the belief arose *a priori*, and on the other hand, from experience simply. The former is the opinion especially associated with the Scottish School of philosophy. Hume holds that it proceeds from a psychological law of custom or habit, — an unbroken line of mental associations inducing a belief within, concerning the uniformity of nature without. Mill has also a like empirical basis for a belief in the uniformity of nature; he holds that having observed uniformity in many experiences, in fact never contradicted, we generalize so as to cover a sphere beyond our experience. Moreover, we possess the consensus of testimony, coextensive with the history of humanity, of the indefinitely wide extent of the sphere of causation, and the accompanying characteristic of uniformity. His position is fortified by the fact that in the process of incomplete induction, its probability is strengthened where there has been exceptionally abundant scope for observation, so that there is the overwhelming conviction that if there had been a time or place where the law would prove untrue, it would have been noticed. Instead of universal causation, Mill and his followers make a more cautious statement; causation as coextensive with the sum total of human experience. This is abundantly adequate to embrace all possible cir-

cumstances of practical inference. The immensely high degree of probability engenders a subjective certitude which in every-day conduct of affairs, and even in the more exact requirements of scientific investigation, is never questioned.

Preyer has given an interesting account of the extremely early appearance of the appreciation of the causal relation in the case of his child, "who, at the three hundred nineteenth day of its life, struck several times with a spoon upon a plate. It happened accidentally, while he was doing this, that he touched the plate with the hand that was free; the sound was dulled, and the child noticed the difference. He now took the spoon in the other hand, struck with it on the plate and dulled the sound again, and so on. In the evening the experiment was renewed with a like result. Evidently the function of causality had emerged in some strength, for it prompted the experiment. The cause of the dulling of the sound by the hand — was it in the hand, or in the plate? The other hand had the same dulling effect, so the cause was not lodged with the one hand. Pretty nearly in this fashion the child must have interpreted his sound-impression and this at a time when he did not know a single word of his later language."[1]

The theoretical soundness of Mill's speculations, however, has a flaw, although the practical results may not be thereby invalidated. The inductive process, which is supposed to be a truly scientific method, and superior to induction by simple enumeration

[1] Preyer, *The Senses and the Will*, pp. 87, 88.

must, according to Mill, at the last analysis, rest upon a principle which is itself based upon an incomplete induction. A very fair and searching criticism of Mill is that of Venn's in his *Empirical Logic.*[1] Whately insists that the whole question concerning the nature of our belief in uniformity is irrelevant, as it is a purely psychological and not a logical one. Mansel holds a mediating position in insisting that the idea of universal causation is intuitive, while that of uniformity is necessarily empirical. Sigwart has very trenchantly criticised Mill in that "taking away with one hand what he gives with the other, he shows in the uncertainty of his views the helplessness of pure empiricism, the impossibility of erecting an edifice of universal propositions on the sand-heap of shifting and isolated facts, or, more accurately, sensations; the endeavor to extract any necessity from a mere sum of facts must be fruitless. The only true point in the whole treatment is one in which Mill as a logician gets the better of Mill as an empiricist; namely, that every inductive inference contains a universal principle; that if it is to be an inference and not merely an association of only subjective validity, the transition from the empirically universal judgment all known *A*'s are *B* to the unconditionally universal all that is *A* is *B*, can only be made by means of a universal major premise, and that only upon condition of this being true are we justified in inferring from the particular known *A*'s to the still unknown *A*'s."[2]

[1] Venn, *Empirical Logic*, p. 130. [2] Sigwart, *Logic*, Vol. II. p. 303.

The whole tendency of the modern logic is to base the causal postulate upon a ground which is epistemological; namely, inasmuch as our knowledge is one and self-consistent throughout all its separate elements, there must be a corresponding invariability in the phenomena themselves, as there is in the system of knowledge which results from the interpretation of these phenomena. This is the general view of Sigwart, Bosanquet, Lotze, and Green.[1]

This view may be considered also as an expression of the Law of Sufficient Reason; namely, that there is an inherent characteristic of intelligence which demands that every element of consciousness must be referred to some other element for its explanation, and that it is only when the logical connection of ideas corresponds to a real causal connection, that the mind discovers a reason for its several experiences which is satisfying. It has been said by Ueberweg, as given expression to this view: "The external invariable connection among sense phenomena is, with logical correctness, explained by an inner conformability to law, according to the analogy of the causal connection perceived in ourselves between volition and its actual accomplishment."[2]

There is a distinction that is of importance to note between the popular and the scientific idea of

[1] Sigwart, *Logic*, Vol. II. pp. 119, 120; Bosanquet, *Logic*, Vol. II. pp. 220, 221; Lotze, *Logic*, p. 68; Green, *Phil. Works*, Vol. II. p. 286.

[2] Ueberweg, *Logic*, pp. 281, 282.

cause. The former is the outcome of the supposition that whatever immediately precedes the effect has evidently produced it, and that this is sufficient wholly to account for it. Such an idea of causes leads, at the best, but to a loose and superficial determination of the relation between any antecedent and its consequent, and there is the danger, moreover, of a hasty inference which results in the fallacy of *post hoc ergo propter hoc.* In order to attain a true view of causation, we must especially attend to the extreme complexity of the causal connection. There is no such thing as a simple cause followed by a simple effect. The cause is always a combination of several elements, circumstances, and conditions; the effect is always manifold. This characteristic has been admirably presented in Mill's chapter on the "Plurality of Causes and the Intermixture of Effects. It is well known that the variation in the height of a barometer is due partly to the variation of the atmospheric pressure, and partly to the variation of the expansion of the mercurial column due to heat. In exact determination, some experiment or calculation must precede, before there can be a discrimination between the elements of the joint effect. And so also, a number of circumstances may combine to restore an invalid to health, no one of which alone being capable of effecting his recovery.

The cause of any phenomenon has been defined by Mill, as also by Brown and Herschel, as the sum total of all its antecedents. This statement has been criticised, inasmuch as the sum total of all

antecedents is indeterminate, and that there is no end to the possible ramifications in all directions which an exhaustive analysis of any complex cause will yield. However, the problem is one of reduction to simplest possible terms within the range of our powers of observation and experiment. There is much in the sum total of all the antecedents of any given effect which is irrelevant. It is the peculiar function of logical analysis to discriminate between the relevant and irrelevant. The temperature of the laboratory will not affect, one way or the other, experiments with falling bodies; but will essentially influence certain chemical experiments, and must enter as one of the determining factors in the sum total of antecedents. It may be that certain elements of a complex whole may seem to us ultimate and unanalyzable, and yet be themselves systems of more or less complexity. There is always a limit to analysis, both experimental and mental. The analysis is to extend to the ultimate parts as far as possible. It is not an exact process, but a process which tends to exactness to the extent which the scope of finite intelligence will permit. The reason is not at fault so much as the natural limitations of observation and experimental analysis. The end of our research in causal analysis is to discover an invariable relation that can be expressed in the form of an hypothetical universal, — If $A$, then $B$.

In order to effect this, the complex $A$ must be separated into its parts, $a$, $b$, $c$, etc., and the effective, and necessary, and indispensable element pro-

ducing $B$ must be determined. Suppose it proves to be $a$, it may be possible to subject this to further analysis, and reduced to simpler elements, such as $x$, $y$, $z$, etc., and $x$ found to be the significant element of the real cause. Each analysis determines a narrower and still narrower sphere within which the cause lies. A man is shot. We say the bullet killed him; then the driving force behind the bullet; then the explosive power of the gunpowder; this in turn was occasioned by the combined chemical and mechanical energy of its ingredients, whereby a solid is transformed into a gaseous substance many times its original bulk.

Sooner or later we must reach the end of our analysis, and the investigation be necessarily checked. No explanation is ultimate; we only transfer our point of view from a less to a more familiar sphere of interpretation. We do not feel the need of explaining the very familiar; though the most familiar is hardest satisfactorily to explain, because there is nothing simpler in whose terms we may paraphrase it. We feel this in giving a definition of terms whose meaning we best know, and which we most frequently use. Mr. Barrett, a former assistant at the Royal Institution, said of Faraday: "I well remember one day when Mr. Faraday was by my side, I happened to be steadying, by means of a magnet, the motion of a magnetic needle under a glass shade. Mr. Faraday suddenly looked most impressively and earnestly, as he said: 'How wonderful and mysterious is that power you have there! The more I think over it, the less I seem to know.'

And yet, he who said this knew more of it than any living man." [1]

Although our knowledge is limited as in all cases of causation, however simple, nevertheless, as far as it goes, the several elements are related logically, that is, necessarily and universally. We may only know in part, but still we know, and the world, as interpreted for us in knowledge, is a world of invariable sequences. The process of inductive analysis, therefore, consists in reducing a complex antecedent to its ultimate parts, in order to reveal the element or elements in it which have caused the given effect. It sometimes happens that different elements in an antecedent may be regarded severally as the cause, according to the psychological point of view as regards the interests of the investigator. It is not always that a scientific determination of the cause is required; it may be that all that is desired is a knowledge of that part of the antecedent which is most closely and prominently connected with the event in question. An inquiry may be started in reference to the cause of an epidemic in a community. One may discover the cause in the carelessness of sanitary engineers; another may say the cause lies in the poor construction of the sewerage; another says that the cause of the epidemic is a certain kind of bacilli. Each one is looking at the chain of events related as cause and effect; but they all look at different links of the same chain. One element, therefore, of a complex antecedent may be brought into more or

[1] Gladstone, *Michael Faraday*, p. 180.

less prominence as the efficient element of the cause, according as the point of view is shifted. If, in the search for the cause of phenomena, the sum total of antecedents were always given exhaustively, the explanation might become so loaded down with details as to burden the mind and confuse, rather than clear, the understanding.

# CHAPTER VI

## The Method of Causal Analysis and Determination

It will be well to consider the various problems which will confront us in seeking to analyze a complex antecedent for the purpose of discovering its cause.

1. There are problems where cause and effect appear in evident sequence. There is an antecedent which is followed by a consequent. If $A$ happens, then $B$ will happen. Instances of this kind most readily yield themselves to the process of analysis, because a change in any given phenomenon is occasioned by the efficiency of the antecedent which is observed in connection with the change itself. It is easier to note active than passive relations, the dynamic rather than the static. The attention is attracted and held by change. The bird flying across our path is observed, and the one perched upon the tree near at hand, however conspicuous may be its position, is passed by without any notice taken of it. It is easier to connect the moisture of the grass with falling rain, than when the same is occasioned by the dew. In one case, the causal relation is ex-

hibited in operation; in the other, the connection is veiled. We find the grass wet; what preceded it we are not able to see. There are several instances of sequence among observed phenomena which must be carefully discriminated in order to avoid confusion of thought. They are as follows: —

(1) When we have $A$ followed by $B$, and $A$ ceases wholly while $B$ endures for an appreciable time afterwards, or it may be permanently. A billiard ball strikes another, the second goes on by virtue of the newly acquired energy transferred by impact from the first, which, however, stops altogether. I throw a ball which lodges on the top of a building; the effect produced lasts permanently, for the ball has gained a gravity potential due to the energy imparted to it by the initial throwing. The old formula, therefore, does not always hold: "Cessante causa cessat effectus."

(2) Cases where $A$ ceases, and thereupon $B$ immediately ceases also. If we cut off the supply of gas which feeds a flame, the flame at once disappears. There are cases, however, when an appreciable time must elapse in order that the transferred energy in the effect may be dissipated. When we shut our eyes the stimulus causing the perception is cut off, and the perception at once is at an end; however, there are cases where the stimulus being very strong, after-images are induced which remain for some time in the dark field after the eyes are closed.

(3) Cases where the antecedent is wholly inadequate to produce the effect, but whose function

is merely to liberate potential energy already stored, and waiting an occasion for its active manifestation. A slight blow upon a piece of dynamite causes an explosion wholly disproportionate to the striking force employed. As is well known, heat is often an exciting cause of chemical action. In such cases the real cause is more or less concealed, while that which is apparent upon the surface is not a cause so much as an occasion of the phenomenon in question. I touch the pendulum and a clock starts and so continues for many hours; the swinging pendulum, however, is only the occasion of liberating the potential energy of the wound-up spring, and thence the power which runs the clock, pendulum, wheels, hands and all.

2. We have also instances not so much of sequence as of concurrence. The planets revolve around the central sun; here the cause is constant, attended by constant effect. The machine never runs down; nor has to be wound up, so that it can be seen that the cause antedates the effect.

3. *Cases of Coexistence.*—These are more difficult to analyze, for the phenomena do not here appear as antecedent and consequent in the midst of changing conditions and circumstances. We have coexistence of two kinds.

(1) Coexisting attributes in one and the same organism. They are always found together. They form one generic concept and are called by one name. Cows have horns, cloven feet, are ruminant, etc. Dogs have their distinct and constant characteristics. The orange has its correlation of

color, taste, smell. And so we have the so-called "natural kinds," *i.e.* organisms presenting an unique and characteristic appearance, differentiated thereby from all others. There are also certain correlations of growth which present a constant relation between certain attributes, as the fact, however we may explain it, that cats with blue eyes are invariably deaf. There are, moreover, illustrations of the same in an inorganic sphere, as the law which connects the atomic weight of substances and their specific heat by an inverse proportion; or that other law which obtains between the specific gravity of substances in the gaseous state, and their atomic weights, they being either equal or the one a multiple of the other. In many cases, the bare fact of coexistence must be accepted without being able to explain the causal ground of it. The several elements present a constant association, and that is all that can be said about it. In other cases, however, a cause may be found as regards, for instance, the correlation of warm-blooded animals always possessing lungs. The connection between respiration and the generation of heat is found to depend upon chemical action as its causal basis.

(2) A relation of statics rather than dynamics, as, for instance, a pillar supporting a roof or arch, is said to be the cause in the sense of the sustaining cause of the superstructure. So also the cohesive force which holds together the particles of a stone. In such cases the energy inherent in the cause is of the nature of a stress and strain.

4. Under this head are embraced the phenomena of vital growth or development. These are the most difficult of all the causal problems to determine; for it is required to discover the inner necessity of essence, and how the succeeding stages of development unfold through the play of the central forces inherent in the very nature and being of the organism itself. Mill is content with classifying organisms as different natural kinds, and he is not concerned with the reason why there should be such and such kinds, nor does he attempt to discover any law concerning these natural correlations and the mode of their growth. In inductive analysis, our concepts must not merely grasp what the natural kinds are, but also what has determined them to be what they are. Darwin puts special emphasis upon the environment as affecting changes in organisms and producing differentiating modifications among species. This, however, must be considered not as sole factor, but one which is combined with inner needs and necessities. Moreover, Darwin has drawn attention to the fact that individual differences need scientific explanation as well as the common attributes, as, for instance, why some sheep are black, and why some pigeons are fan-tailed and others are not. In all such considerations we must not lose sight of the fact that there are two determining factors, — the inner necessity of development; and the external necessity of causality, as organisms are acted upon by their environment.[1]

5. Cases of collocation where no one element of

[1] Sigwart, *Logic*, Vol. II. pp. 322, 330, 331.

the cause is efficient, but all together they combine to produce the effect. In searching for the cause, we must not only find a certain amount of energy capable of producing the effect, but we must also discover what peculiar arrangement of the elements concerned must exist before the energy in question can become operative. Chalmers says that "the existing collocations of the material world are as important as the laws which the objects obey, that many overlook this distinction and forget that mere laws without collocations would have afforded no security against a turbid and disorderly chaos."[1] We would naturally say that the sole cause of water boiling at 212° is the enveloping heat; it has, however, been observed that on top of Mont Blanc, water boils at 180° instead of 212°. This indicates that, in addition to the fire, the atmospheric pressure is an element in the cause, very easily overlooked. Charcoal and diamond are of the same substance; a difference only in the arrangement of the molecules results in such radically different combinations. There are, in the main, three special kinds of collocations, as follows: —

(1) Cases of modifying circumstance. A strong wind blows down a tree; this would not have occurred had not the tree been hollow. The hollowness of the tree is here a co-operative circumstance that is combined with the efficient cause, — the force of the wind. An instance where arrangement of the elements concerned rather than their efficient energies is productive of the effect, is that of capil-

[1] Quoted by Jevons, *Principles of Science*, p. 740.

larity, the rising of liquid in a tube of exceedingly small bore. Here form is more essential to the effect than the expenditure of any visible energy.

(2) Cases in which certain negative conditions prevent the realization of the effect. The plants and shrubs die in a long drouth, because it did not rain. A train collides with another, because the red signal was not exposed as it should have been. A match will ignite gunpowder generally, but it fails to do so should the powder prove to be wet.

(3) There are also cases of counteracting causes, where the effect of cause *A* is not realized, as cause *B* neutralizes the force of cause *A*; as when an anchored boat will not respond to the pull of the oar. Sometimes the cause is not wholly counteracted, or it may be the counteracting cause more than holds the positive cause in check, and is itself operative. The rise of a balloon in the air is due to the fact that the force of gravity is more than overbalanced by the expansive force of the gas within the balloon; one force pulling downwards, the other bearing up, and the latter prevailing.

Mechanical forces acting in combination admit of a resolution of their joint effect according to the theory of the parallelogram of forces. Chemical and vital forces cannot be treated in such a way at all. From the character of the elementary forces in mechanics, one can calculate their combination. In chemistry, however, when the elements are given, the resulting compound cannot be thus determined. So, also, in vital and mental phenomena, the necessarily complex nature of the

elements involved prevents not only prediction of resulting combinations, but even adequate explanation of that which may be immediately given in consciousness.

6. It is necessary, in the investigation of causal relations, to understand the different modes of the transfer of energy, which are as follows: —

(1) Molar or mechanical, as in the case of a billiard-ball transferring its energy to another through impact.

(2) Molecular, as heat, chemical and electrical and magnetic forces, light, etc. One passes into another, as chemical force producing electric, electric producing magnetic, or producing heat and light.

(3) Cases where mechanical force becomes molecular, as friction inducing heat; or cases where molecular becomes mechanical, as heat transferred into the driving power of an engine, or electricity applied as a motor. A precise determination of equivalents can be made between molar and molecular energy; as, for example, it has been found that it takes the same amount of energy to raise 772 pounds a distance of one foot that it does to raise the temperature of one pound of water 1° F.; or the heat requisite to boil a gallon of freezing water would lift 1,389,600 pounds through a distance of one foot.

As a consequence of the doctrine of the transfer of energy, a causal law can be so stated as to express the fact that variations in the antecedents will call for corresponding variations in the effect,

as, for instance, such a law as the following: "Resistance in a wire of constant section and material is directly proportional to the length and inversely proportional to the area of the cross-section."[1] The neglect of quantitative determination of the proportionate variations of the antecedent and consequent was a glaring defect in the inductive systems both of Mill and of Bacon.

Through the representation of the various stages of such variation, it is also possible to establish the upper and lower limits beyond which the cause does not produce the corresponding effect; as in Weber's law concerning the relation of stimulus to sensation, that stimulus must increase geometrically in order that the sensations increase arithmetically. There is an upper and lower limit beyond which this proportion does not hold.

The doctrine of conservation of energy creates the impression of continuous change in causation, in which the effect unfolds out of the cause. We do not think of phenomena under this aspect as discrete events. More than ever, in the light of modern science, does the old saying obtain, "Natura non facit saltum." We no longer look for catastrophic results in nature — but regard causation as a continuous transfer of potential energy into kinetic or actual energy.

We come now to the consideration of the method by which the causal analysis is mediated. This is effected through observation and experiment. Observation is something more than mere looking at

[1] Jenkin, *Electricity and Magnetism*, p. 83.

phenomena; it means concentration of attention for the purpose of research; it means discriminating insight, an appreciation of likeness and difference; it means a penetration beneath surface appearances, and an apprehension of the essential features of the objects of perception. Experiment consists in modifying the elements which form the complex antecedent in order to observe the resultant effect upon the corresponding consequent. Forces may be added or subtracted; their intensity may be varied, increased, or decreased; the circumstances or conditions may be altered. Herschel speaks of observation and experiment as passive and active observation. When we interfere to change the course of nature, or to bring natural forces within the range of our observation, we are experimenting. Observation is preliminary to experiment, and suggests the lines along which experiment should proceed. An observation that sees the parts in the whole and the whole in the parts, is in itself an analysis of a phenomenon, in course of which process causal relations must be disclosed. The scientific spirit demands absolute veracity in observation. One ought not to be blind to facts even though they tend to contradict preconceived theories. Bacon has observed that "men mark when they hit, never mark when they miss." We must strive against a natural tendency to see things as we would have them, and not as they strictly are.

We must also carefully distinguish between observed facts, and inferences which we instinctively draw from these facts. Observation is preliminary

to an inductive inference, therefore it must not itself involve an inference, or we should be arguing in a circle. An interesting illustration of the difference between observation and inference based upon it, is narrated in the life of Faraday: "An artist was once maintaining that in natural appearances and in pictures, up and down, and high and low, were fixed indubitable realities; but Faraday told him that they were merely conventional acceptations, based on standards often arbitrary. The disputant could not be convinced that ideas which he had hitherto never doubted, had such shifting foundations. 'Well,' said Faraday, 'hold a walking-stick between your chin and great toe; look along it and say which is the upper end.' The experiment was tried, and the artist found his idea of perspective at complete variance with his sense of reality; either end of the stick might be called upper, — pictorially it was one, physically it was the other."[1]

This indicates how readily our inferences and observations blend, and how difficult it is to separate them in consciousness. De Morgan has pointed out that there are four ways of one event seeming to follow another, or to be connected with it, without really being so: —

(1) Instead of $A$ causing $B$, our perception of $A$ may cause $B$. A man dies on a certain day which he has always regarded as his last through his own fears concerning it.

(2) The event $A$ may make our perception of $B$ follow, which would otherwise happen without

[1] Gladstone, *Michael Faraday*, pp. 165, 166.

being perceived. It was thought that more comets appeared in hot than cold summers; no account, however, was taken of the fact that hot summers would be comparatively cloudless, and afford better opportunities for the discovery of comets.

(3) Our perception of *A* may make our perception of *B* follow. This is illustrated by the fallacy of the moon's influence in the dissipation of clouds. When the sky is densely clouded, the moon would not be visible at all; it would be necessary for us to see the full moon in order that our attention should be strongly drawn to the fact, and this would happen most often on those nights when the sky is cloudless.

(4) *B* is really the antecedent event, but our perception of *A*, which is a consequence of *B*, may be necessary to bring about our perception of *B*. Upward and downward currents are continually circulating in the lowest stratum of the atmosphere; but there is no evidence of this, until we perceive cumulous clouds, which are the consequence of such currents.[1]

There are certain natural limitations to observation, as things too minute to be seen, too swift to be carefully examined; there are sounds which some ears can detect, while others cannot, and shades that some eyes cannot discriminate. There are effects proceeding from certain causes that are so slight that we fail to observe them, and yet erroneously infer that they do not exist. Professor Tyndall has given a striking illustration of the dif-

[1] Quoted by Jevons, *Principles of Science*, pp. 409–411.

ference of auditory power in two individuals; he says: "In crossing the Wengern Alp in company with a friend, the grass at each side of the path swarmed with insects which to me rent the air with their shrill chirruping. My friend heard nothing of this, the insect music lying quite beyond his limit of audition."[1] Much has been done by inventive skill to increase our powers of observation, and at the same time to render them more accurate, as the telescope, microscope, the vernier for precise measurement of minute differences of magnitude, the chronograph for time measurements, self-registering thermometers, the thermopile, galvanometers, etc. One of the chief problems of scientific method is to overcome natural limitations of observation through mechanical devices.

Observations on a large scale and over a considerable period of time must sometimes be taken in order to disclose tendencies as seen only in the average or the mean of the observed results. Thus meteorological, vital statistics, and others of a like kind must extend over a large area, and embrace a large number of instances in order to reach results of any value. It is known that Tycho Brahé made an immense number of most exact records of the positions of the heavenly bodies with the aid of the best of astronomical instruments, and these records afterwards became the foundation of Kepler's laws and of modern astronomy.[2]

The faculty for accurate observation can be in-

[1] Tyndall, *On Sound*, pp. 73, 74.

[2] Gore, *The Art of Scientific Discovery*, p. 316.

creased by acquiring the habit of examining carefully everything within the field of vision. We fail to see many things because we fall into the easy way of passing them by without noting their presence or appreciating their significance. It was said of Charles Darwin by his son that "he wished to learn as much as possible from every experiment, so that he did not confine himself to observing the single point to which the experiment was directed, and his power of seeing a number of other things was wonderful."[1] The open-eyed vision is the prime requisite for scientific investigation.

The limitations of observation naturally lead to experiment, whose special function is to so modify phenomena as to bring a suspected causal element more prominently into notice. This can be done by intensifying the force in question, or by neutralizing all other elements in combination with it, so that the sole effect of this force in actual operation can be observed. When the cause is not a simple element, but a combination, then the problem is to vary the conditions so that but one possible combination, then another, can be operative alone, and note the corresponding effect. Given a certain number of elements, the number of possible combinations is mathematically determinate, and can be tried *seriatim* until all possibilities are exhausted. Venn has given a long and interesting illustration of this in his *Empirical Logic*.[2] All combinations need not be tried, however; for many will be seen

[1] *Life and Letters of Charles Darwin*, Vol. I. p. 122.
[2] pp. 402 ff.

to be either impossible or irrelevant. The aim is to obtain an antecedent which shall consist either of a simple element, or a combination such that with its presence the effect in question is present also, but with its disappearance the effect is wanting.

It is not sufficient to note merely the presence of an antecedent connected with a corresponding consequent; scientific determination consists, in addition, in proving the absence of the suspected cause in cases where the given effect is not present. This is called determination by negation. A proposition which is held affirmatively has only a vague significance; it must be determined within definite limits assigned to it by virtue of what it is not. Defining means to set limits to a term; these limits grow out of the nature of the thing itself. The negative judgment marks a transition always from that which is indefinite and incoherent to that which is definite and coherent. For instance, we have a vague notion of chemical affinity that elements combine to form compounds. That is the nucleus of our knowledge; it grows in definiteness through a continuous process of limitation by negation. We find that not all elements combine with each other, that they do not combine except in certain proportions, and that even those which do in certain definite proportions will not combine in the presence of others having greater affinity, as, for instance, in the presence of oxygen, and so on. Every negative proposition established renders the original one more accurate.

This may be illustrated also in the concrete, when

in dissection one is tracing a nerve; it is followed throughout its course by a series of negative judgments though they be unexpressed: This is not a nerve, but an artery; this is not a nerve, but a vein; this is not a nerve, but a filament, or shred of muscle, etc. So we rise through negative discrimination to a clear apprehension of an object under investigation. The original proposition must be readjusted with every new negative determination. It sometimes happens that the original proposition is completely negatived by the negative determination, sometimes again it is confirmed.

A proposition that has not been worked over through such a process has no real logical worth or scientific value. Therefore in the analysis of phenomena when the suspected cause and effect are combined in a proposition, it can at first be held only tentatively. It must be confirmed negatively, or else readjusted to conform to the negative requirements. Suppose we have given that $A$ is followed by $B$ as far as we have been able to observe. We may proceed by experiment to multiply instances of $A$'s connection with $B$, but still the causal relation is not absolutely proved. We must go on to show that in all cases of not-$A$ there is not-$B$, or in all cases of not-$B$ there is not-$A$. Negative experiment produces the contrapositive, or the converse contrapositive of the proposition under investigation, which deductively necessitates the validity of the original proposition.

This is substantially Mill's method of difference, that if an instance in which the phenomenon under

investigation occurs, and an instance in which it does not occur, have every circumstance save one in common, and that one occurring only in the former; the circumstance in which alone the two instances differ, is the effect or cause or a necessary part of the cause of the phenomenon. This method will be described later; it is the main inductive method, the others being largely modifications of it. A negative instance which is established concerning relations of not-$A$ and not-$B$, is absolutely conclusive, inasmuch as not-$A$ is the contradictory of $A$, and not-$B$ is the contradictory of $B$. They are mutually exclusive. No other possibility can be forthcoming, and the experimental analysis is exhaustive. Professor Tyndall gives the following account of an experiment to determine the cause of resonance. "I hold a vibrating tuning-fork over a glass jar eighteen inches deep; but you fail to hear the sound of the fork. Preserving the fork in its position, I pour water with the least possible noise into the jar. The column of air underneath the fork becomes shorter as the water rises. The sound augments in intensity, and when the water reaches a certain level, it bursts forth with extraordinary power. I continue to pour in water, the sound sinks, and becomes finally as inaudible as at first." [1]

From this it is inferred that a certain column of water of definite height is necessary to the production of the sound, for above and below the limits no sound is heard. This experiment also indicates that which is most important in causal determina-

[1] Tyndall, *On Sound*, p. 172.

tion, — a variation in cause accompanied by a variation in effect, as also a maximum and minimum as regards the intensity of the sound. Experiment proceeds upon the supposition of the measurableness of phenomena, and seeks numerically expressible results in this regard. For instance, by different experiments, Tyndall proved that the length of the column of air which resounds to the fork in a maximum degree of intensity is equal to one-fourth of the length of the wave produced by the fork.[1]

The negative determination of a suspected connection of cause and effect must be precise in order to establish the causal relation with that degree of accuracy which is demanded in a truly logical and scientific method. Upon this point, Bosanquet has a very suggestive passage: "The essence of significant negation consists in correcting and confirming our judgment of the nature of a positive phenomenon by showing that *just when* its condition ceases, *just then* something else begins. The 'Just-not' is the important point, and this is only given by a positive negation within a definite system. You want to explain or define the case in which $A$ becomes $B$. You want observation of not-$B$, so that you are lost in chaos. What you must do is to find the point within $A$ where $A_1$ which is $B$, passes into $A_2$ which is $C$, and that will give you the *just-not-B* which is the valuable negative instance."[2] For example, in Professor Tyndall's experiment, the

[1] Tyndall, *On Sound*, p. 174.

[2] Bosanquet, *The Essentials of Logic*, p. 134.

significant negative instance was this, — when the water in the tube reached just that height when for the first time during the experiment no sound was audible. The discriminating observation that can mark and measure the precise point of transition from sound to no sound, has determined accurately the conditions necessary to produce the sound, and precisely define their limitations.

In all observation and experiment, the following possibilities should be kept before the mind in order to avoid a hasty conclusion in reference to a seeming causal connection. We may think that we have discovered the relation that if there is $A$, then there must be $B$, and the one therefore the cause of the other, but it may happen that

1. Both $A$ and $B$ are effects of another cause and are thereby related co-ordinately in reference to it.

2. $A$ may be merely a liberating circumstance, or an invariable accompaniment of $B$.

3. $A$ may not be the cause of $B$, but only an element of a complex collocation which is the cause of $B$.

4. Each separate instance of $B$ may so differ as to respond to the action of $A$ in a manner different from the others.

5. $A$ may be related to $B$ in a system of such a nature, that the system in continuously developing new effects causes $B$, as the introduction of medicine into an organism whose forces are themselves effecting a healing process.

6. It is often very difficult to tell whether $A$ is the cause of $B$, or $B$ the cause of $A$, as in districts

where drunkenness and poverty are prevalent, or cases of moral and intellectual feebleness. Which is the cause? and which the effect? In many cases such as these, the forces react upon each other, the effect tending to increase the intensity of the cause.

7. The connection of *A* and *B* may be one of mere coincidence, and not of a causal nature whatsoever. Newton was much impressed with the apparent connection between the seven intervals of the octave, and the fact that the colors of the spectrum divide into a like series of seven intervals. And yet there is no causal connection that can be proved to exist between the two.

The more we dwell upon these various possibilities, the more are we impressed with the extreme complexity in which the relation of cause and effect is involved. The investigator must bring to his research the spirit of patience and perseverance, as well as a clear vision and discriminating insight. Sir John Lubbock, in his observations upon the habits of ants, says that at one time he watched an ant from six in the morning until a quarter to ten at night, as she worked without intermission during all that time.[1] It is to such patient investigators that Nature reveals her secrets.

[1] Sir John Lubbock, *Scientific Lectures*, p. 73.

# CHAPTER VII

## MILL'S INDUCTIVE METHODS — THE METHOD OF AGREEMENT

THERE are various methods of causal research which have received the name of inductive methods and have been especially associated with the contribution of John Stuart Mill to the history of logic. There are five of these methods or inferential processes as given by Mill, and forming the integral part of his system of induction. They are as follows: —

1. The Method of Agreement.
2. The Method of Difference.
3. The Joint Method of Agreement and Difference.
4. The Method of Concomitant Variations.
5. The Method of Residues.

The method of agreement consists in inferring the existence of a causal relation, when in a number of varying instances it is observed that the supposed cause is always accompanied by the phenomenon in question, as corresponding effect. The method of difference is the comparing of an instance where the supposed cause is present, accompanied by the corresponding effect, with an instance

having precisely the same setting, but where the supposed cause is withdrawn, the effect also disappearing; the inference of a causal relation is then permissible. The joint method of agreement and difference is the comparing of instances where the supposed cause is present, with similar instances where it is absent; if the corresponding effect is present in the former, and absent in the latter group of instances, a causal relation may be inferred. This differs from the method of difference, that in the latter the same instance, now with, and again without the presence of the suspected cause, is the subject of observation; in the joint method it is a number of instances with, compared with a number of similar instances without, the presence of the supposed cause. The method of concomitant variations consists in so modifying any given phenomenon that the supposed cause will vary in intensity; then a corresponding variation in the accompanying effect is evidence of a causal relation. The method of residues consists in the analysis of a given complex phenomenon, in which all elements save one of the antecedent are known to be related in a causal manner to all elements save one of the consequent; then the residual element of the one may be regarded as the cause of the residual element of the other.

We will now examine these methods more in detail. The brief outline above is intended merely to give a general idea of the methods, that it may lead to a better understanding of the more exact statement of their nature and characteristics.

*The Method of Agreement.* — The more precise statement of this method is given in the first canon of Mill, which is substantially as follows: —

If two or more instances of the phenomenon under investigation have only one circumstance in common, the circumstance in which alone all the instances agree is the probable cause (or effect) of the given phenomenon, or sustains some causal relation to it.

The above is based upon the causal axiom that the constant elements which emerge in any given series of similar phenomena are to be considered as connected in some manner with the cause of the phenomena; but that the variable elements are not connected with the phenomena in any causal manner whatsoever.

The method of agreement is illustrated in the investigation of the very common phenomenon of the transformation of substances from the solid to the liquid state. What is the one circumstance which is always present when we consider the melting of such widely different substances as butter, ice, lead, iron, etc.? In all instances, to whatsoever extent they may be multiplied, of the change from solid to liquid states, heat has been observed to be present, and is thereby indicated as the likely cause of the phenomenon in question. The method may be represented through the use of symbols which, according to Mill, are the capital letters to denote antecedents, and the smaller letters to denote corresponding consequents. Let the following be a number of different instances with the antecedents

and consequents arranged in order, and represented as above indicated: —

| | | |
|---|---|---|
| $ABC$ | . . . . . . . . . . | $abc$ |
| $ADE$ | . . . . . . . . . . | $ade$ |
| $AMN$ | . . . . . . . . . . | $amn$ |
| etc. | | etc. |

By inspection of such a table of instances thus analyzed, and symbolically represented, it will be readily seen that $A$ is the only element common to all the antecedents, while $a$ is the only one common to all the consequents. The inference, therefore, is that $A$ is the cause of $a$. It has been objected to this system of representation that it artificially arranges the elements of antecedent and consequent, as though there were a number of distinct cause-elements, each connected with a correspondingly distinct effect-element, and it produces the impression that it is quite an easy matter to see how these causal pairs are thus separately related.[1] As nature presents her phenomena to us, however, there is such complexity throughout, that the analysis cannot readily distribute part to part in appropriate causal relations. To avoid such an error in notation, I have adopted the following symbols, which will be used hereafter to describe the various methods. Let us take $C$ as the letter to represent the supposed causal element, and $S$, the entire setting of accompanying circumstances; let $e$ denote the corresponding effect, and $s$ the sum total of the attendant consequences. The causal relation will

[1] Venn, *Empirical Logic*, p. 411.

be then indicated, according to the method of agreement, as follows: —

$$\begin{array}{lcl} S + C & \ldots\ldots\ldots\ldots & s + e \\ S' + C & \ldots\ldots\ldots\ldots & s' + e \\ S'' + C & \ldots\ldots\ldots\ldots & s'' + e \end{array}$$

Here the setting changes throughout, as indicated by $S$, $S'$, $S''$, etc., but $C$ remains constant in the antecedents; also the corresponding setting in the consequents changes, as indicated by $s$, $s'$, $s''$, etc., but $e$ remains constant throughout. Such a notation does not attempt to represent just which parts of $S$ cause corresponding parts of $s$, nor by what elements precisely $S$ differs from $S'$ and $S''$, etc. It does represent, however, the difference between the variable and constant elements of the table of instances which are arranged for comparison, and this is the key to disclose the causal relation.

As an example of this method, let us take the physical law that different bodies tend at the same time to absorb and to emit the same waves of light. It is known that every substance in burning gives its own lines in the spectrum analysis, sodium, for instance, producing a very bright line in the yellow portion of the spectrum in a definite locality (Line $D$, of Fraunhofer). If now, instead of burning sodium, we interpose the vapor of sodium in the path of the ray which should give a continuous spectrum, the phenomenon is completely reversed; at the exact point where there was a bright line in the spectrum, a dark line now appears. Thus the vapor of sodium, acting as a screen, absorbs the

rays which it emits when it acts as the luminous source. A similar effect is observed in the case of vapors of iodine, of strontium, of iron, etc.; and is a phenomenon, therefore, admitting of generalization by induction.[1] This is according to the method of agreement; and we may make the following representation: —

| | | | | | |
|---|---|---|---|---|---|
| Vapor of | sodium | acting as a | screen | = | $S + C$ |
| " | iodine | " | " | " | $= S' + C$ |
| " | iron | " | " | " | $= S'' + C$ |
| " | strontium | " | " | " | $= S''' + C$ |
| | etc. | | | | etc. |

The corresponding consequents are: —

| | | | | | |
|---|---|---|---|---|---|
| Reversing | bright | sodium | line to | dark | $= S + e$ |
| " | " | iodine | " | " | $= S' + e$ |
| " | " | iron | " | " | $= S'' + e$ |
| " | " | strontium | " | " | $= S''' + e$ |
| | | etc. | | | etc. |

Therefore we have: —

$S + C$ . . . . . . . . . $s + e$
$S' + C$ . . . . . . . . . $s' + e$
$S'' + C$ . . . . . . . . . $s'' + e$
$S''' + C$ . . . . . . . . . $s''' + e$
etc. etc.

In this the constant $C$ of the antecedents is the vapor of any substance acting as a screen; the constant $e$ is the reversal in each case of the bright

[1] Saigey, *The Unity of Natural Phenomena*, pp. 94, 95.

line of the substance in the spectrum to the corresponding dark line of the same. From this it is inferred that the vapor of any substance acting as a screen absorbs exactly those rays which it emits when it acts as the luminous source.

It is of great importance that the instances selected for observation or experiment be as varied as possible, so that widely differing phenomena may be gathered together. Then if running through them all there is one common element observed among the antecedents, and one common element among the consequents, the greater the variation among the instances the more pronounced will be the significance of the constant elements. In the illustration given the substances which are so different as iron, strontium, sodium, iodine, etc., preclude the possibility of the resultant phenomenon described being due to the peculiar properties of any one metal, or group of metals. So many, and so different in kind, are taken as to eliminate the peculiarities attached to any one in particular. In this respect, the method is one of elimination. By varying the instances, the non-essential is eliminated, and the essential, which remains as the element common to all, is thereby emphasized, and differentiated from all attendant circumstances.

This method also is one of discrimination, of discerning the constant element under the various changing forms which it can assume, and as such it is similar to the logical process of the formation of a concept. The concept is the grasping of the universal element which is present through the

particular and concrete manifestations of the same. Through them all there is the like common element which is the basis of the concept itself. So out of many particular instances the mind grasps the elements which are common to all, and considers them as related in a constant and therefore causal manner, which has in itself the character of a universal concept and so admits of being formulated in the form of a law universal, which is the end of all induction.

This method, moreover, is peculiarly adapted to observation, the collating of a number of instances, rather than to experiment. Instances cannot always be manufactured, and so it may be beyond the power of experiment to reproduce them. They can, however, always be the objects of research, and as such fall naturally into the field of observation.

The question may properly be asked at this point, How does this method differ from that of induction by simple enumeration? The latter we have seen is never satisfactory because the enumeration cannot be complete, and may be contradicted by an enlarged experience. This method, however, is superior in that it provides for more than simple enumeration of instances in which the phenomenon in question has occurred; there must be a corresponding analysis of the instances, accompanied by a discriminating insight to distinguish the essential from the unessential. Number of instances increases the probability that the variable elements have been eliminated, and enables the mind to con-

centrate upon the constant elements that remain and are thereby disclosed.

This method primarily admits of application to instances where a sequence is observable; that is, where antecedent can be distinguished from consequent by an appreciable time element. It is, however, possible to apply this method to the investigation of coexistences, where it may show that either the coexisting elements are related as cause and effect, or that in some causal manner they are the correlated effect of some cause sufficient to account for them both. Many instances may be adduced of the prevalence of poverty and crime associated together. This may indicate a causal relation between them, and yet a sequence cannot be observed of sufficient definiteness to indicate which is the cause, and which the effect. The problem is thus left indeterminate, with the suggestion of some other cause which may possibly account for them both. All that the method of agreement can attain, is by collecting a number of instances of diverse nature to indicate that in some way at least poverty and crime are connected by causal ties. The constant coexistence of attributes in one individual admits of a similar treatment and similar results. The fact of the high coloring of male butterflies in a large number of instances, in reference to a variety of species, indicates a constant relation between the fact of its being a male and the possession of brilliant coloring. This inseparable association indicates a causal relation, which, however, cannot be more precisely determined by this method. The

full explanation of the phenomenon requires some supplementary hypothesis depending upon conditions not disclosed by this method, an hypothesis such that the high coloring has the special function of attracting the female butterfly and has been intensified and developed by natural selection.

The method of agreement is open to criticism at several points, and yet it must be at the beginning understood that this method does not rank as a final method. We shall soon see that it serves rather as suggestive of and leading to experiments according to the method of difference, to corroborate or disprove the results which the method of agreement may have attained. The chief criticisms that have been made of this method may be summed up as follows: —

1. The cause indicated by the method of agreement is not thereby proved to be the sole cause of the phenomenon in question. We may gather together a number of varied instances where an extensive failure of crops in the summer has caused hard times during the winter following. And yet there may be, and as a fact there are, many other causes which engender periods of industrial depression. We may say, therefore, that this method is capable of establishing, tentatively at least, an universal proposition of the form, All $x$ is $y$; it does not, however, attempt to give any indication one way or the other, regarding the validity of the converse, All $y$ is $x$. Knowing the limitations of a method, does not by any means destroy its legitimacy as a method; it rather increases its efficiency within its

proper sphere, by the more exact knowledge as to the precise extent of that sphere itself.

2. It is urged that while it is possible to recognize in most, if not in all cases, the common element in the several effects of similar phenomena, it is not so easy a matter to differentiate the common element in the corresponding antecedents by the simple method of agreement alone. For instance, in Bacon's illustration of the investigation of the cause of heat, he cites such disparate phenomena as the sun's rays, friction, combustion, etc. The element of heat is readily discernible through them all; but what is the common element which operates as cause in each case? There is the difficulty. Sigwart illustrates this in the case of the phenomenon of death. The effect can be easily detected as similar throughout, but in all the antecedents the only property common to them all is life, and, therefore, we are led into the fallacy of attributing to life the cause of death.[1] We must therefore acknowledge that some phenomena may occur in such a variety and such a number of manifestations as to disguise the nature of the cause under the mask of a generality too indefinite to be recognized. In all such instances, the method of agreement must operate upon suggestions received from some other source, as to the nature of the common element in the antecedents. Or, some minor circumstances attending the effect may indicate more precisely the nature of the cause, as, for instance, the peculiar symptoms associated with death by drowning.

[1] Sigwart, *Logic*, Vol. II. p. 341.

3. The common element in the antecedents may prove to be an unessential accompaniment of all the instances examined. Its presence, therefore, may have nothing whatsoever to do with the observed effects. A number of different medicines, for example, may produce a certain effect alike in all instances. The only common element that can be detected in the various medicines examined, may be the alcohol which is used as the common vehicle of the different drugs, and yet its effect may be entirely inert as regards the medicinal qualities in question. The common element really efficient may be overlooked, and another common element which is easily discernible may nevertheless remain wholly inoperative. This difficulty may be overcome by a more thorough analysis of the phenomena observed, which may be attained by a judicious variation of the instances, so as to reveal, in turn, the precise effect of the various simple elements which together constitute the complex whole of the phenomenon in question. The defects of the method in this respect are, in a word, the defects of induction by simple enumeration.

4. The cause may be present in all the antecedents, and, notwithstanding the corresponding effect, not appear, and this, not because the two are not related in a causal manner, but because the cause is neutralized by the associated elements which appear in combination with it in the various antecedents. For instance, diphtheria germs are the cause of diphtheria, and have been found accompanying this disease in all cases which have been ob-

served. And yet their presence is often noted when the disease itself does not develop. The tendency existing is counteracted by the condition of the organism at the time, so that the dread bacilli are inoperative and therefore harmless. As we have seen before, the presence of the effect necessitates the presence of the corresponding cause; but by no means is it always true, that the presence of the cause necessitates the effect. The cause always produces the tendency at least, which, however, may be neutralized.

5. This method is often applied in a very careless way to the observations of persons who do not possess the power of accurate discrimination, and therefore observed coincidences are hastily assumed to be particular instances of an universal law. Such procedure leads to superstition and prejudice. It not only warps the judgment, owing to its illogical nature, but it also affects indirectly the man's moral view, as it implies a weakness in character as well as in mind. This criticism refers, however, to the abuse rather than the legitimate use of this method under such restrictions as have been already indicated.

The chief function of this method is that of suggestion. It indicates often only the possibility of the existence of a causal relation; in other cases it leads to an inference of high probability. In all cases, however, it marks but the preliminary steps of an investigation which should be followed up by painstaking experiment. As it is the method of observation chiefly, it is natural that it should pre-

cede experiment; for it is only by reflection upon our observations that we discover the nature and relations of phenomena, which serve as data for subsequent experiment.

I have selected several illustrations to indicate the various fields of research in which this method of agreement has led to satisfactory results.

The first refers to the relation between the occurrence of financial crises and the prevalence of over-production. Guyot, in his *Principles of Social Economy*, gives the following instances: An enormous consumption of capital in the United States in the seventies for the construction of railroads, was followed by unusual commercial depression. Then the like outlay in India for railway construction by means of loans and taxes which absorbed the whole circulating capital of the Indian population, was followed by a devastating famine and general commercial paralysis. Again in Germany there was an enormous consumption of capital in forts and armaments and general military equipment, bringing on the crisis of 1876–1879. England at the same time was unduly supplying circulating capital to the United States, Egypt, and her colonies, and a financial crisis was the result. Through all these varying instances and others of a like nature which might be added, the constant relation of over-consumption in the antecedents to the industrial depression evident in the effect, indicates the one to be the cause of the other, either in whole or in part.

Again, it is narrated in Brewster's *Treatise on*

*Optics* that he accidentally took an impression from a piece of mother-of-pearl in a cement of resin and beeswax, and, finding the colors repeated upon the surface of the wax, he proceeded to take other impressions in balsam, fusible metal, lead, gum arabic, isinglass, etc., and always found the iridescent colors the same. His inference was that the form of the surface is the real cause of such color effects.[1] The common element which appears in all the antecedents is evidently the same form impressed upon each, which was originally received from the mother-of-pearl. The cause is, moreover, independent of the nature of the substance in each case which received the impression upon its surface, because such a variety of substances was chosen as to eliminate the individual nature of each as an influencing factor in the result. In this experiment we see the advantage of varying the instances as far as possible for this very purpose of eliminating all irrelevant elements. Similar experiments have proved like results in reference to the colors exhibited by thin plates and films. Here the rings and lines of color have been found to be nearly the same whatever may be the nature of the substance. A slight variation in color is due to the refractive index of the intervening substance. With the exception of this, the nature of the substance is not operative in producing the color effect, but the form alone.

The celebrated scientist, Pasteur, in the year 1868 was carrying on his investigations as to the

[1] Quoted by Jevons, *Principles of Science*, p. 419.

cause of the blight then devastating the silkworms of France. One of his experiments consisted in selecting thirty perfectly healthy worms from moths that were entirely free from the corpuscles, which latter are the germs of disease, or at that time suspected to be the germs of disease. Then, rubbing a small corpusculous worm in water, he smeared the mixture over the mulberry leaves. Assuring himself that the leaves had been eaten, he watched the consequences day by day. One after the other the worms languished; all showed evidences of being the prey of the corpusculous matter, and finally, within one month's time, all died. Pasteur's inference naturally was that the corpuscles had produced the death. Of course his results were not founded upon this experiment alone, but other experiments, carried on in many different ways, served to corroborate the causal relation which the experiment just described had suggested as at least highly probable.

In medicine also the method of agreement is often used with effect. Certain drugs are administered in a number of cases and the results noted. An uniform effect consequent upon the administration of a given drug indicates a causal connection capable of generalization. Not only are subjects in disease, but also in health, selected, and the effects upon both the normal and morbid natures compared. Thus a variation in instances is secured. If a number of different drugs produce like effects, the question at once suggests itself, What is the property common to them all? The method of agreement

often gives some indication of this, when the elimination of the inert properties can be accomplished through a sufficient variation of instances. The difficulty lies, however, in this very thing, to so vary the instances as to disclose the efficient element present in them all. Various medicines present a complex nature of such a character that it is extremely difficult to ascribe the precise effects which the several component parts individually exercise.

The method of agreement is also used, perhaps unconsciously, in the conduct of the every-day affairs of life. Whenever different phenomena in our experience present certain characteristics of a constant nature, we are at once led to suspect a causal connection, and to start upon a more searching investigation of the same. Too often, however, the supplementary investigation is omitted, and the mind rests content with a few surface resemblances that lead to a hasty generalization, without being more precisely and adequately determined.

# CHAPTER VIII

## The Method of Difference

The method of agreement, as we have seen, presents a causal relation as a suggestion, admitting of a high degree of probability it may be, but requiring to be tested by some more scientific method. This is accomplished by the method of difference. Here a phenomenon is observed, in which the supposed cause-element and effect-element appear; then while all other circumstances and conditions remain unaltered, the supposed cause-element is withdrawn, or its force adequately eliminated; the immediate disappearance of the supposed effect-element consequent upon this, indicates a causal relation existing between the two. Or the experiment may be made in a different manner, but to the same end; that is, a phenomenon may be characterized by the absence of both cause-element and effect-element; then, if the introduction of the cause-element does not disturb the phenomenon in question, except immediately to produce the effect-element, the inference may be drawn that the one is the veritable cause of the other.

*Canon of the Method of Difference.* — If an instance in which the phenomenon under investigation

occurs, and an instance in which it does not occur, have every circumstance, save one, in common, that one occurring only in the former, the circumstance in which alone the two instances differ is the effect, or it may be the cause, or a necessary part of the cause, of the phenomenon.

This method has manifold illustration in our every-day inferences. A person is asleep in the room with us, and we hear the loud noise of a slamming door, and observe the person at once awakening with a start and exclamation. We have no hesitancy in ascribing the awakening to the noise immediately preceding it. We observe again some one receiving a letter or telegram, and immediately upon opening it the face grows white with anxiety and fear, the hands tremble, and there are shown general symptoms of perturbation. The message received, we say, has caused the mental shock and physical accompaniments.

Or, taking a simple experiment in quite another sphere, it was observed by Boyle, in 1670, that an extract of litmus was immediately turned red by the introduction of an acid. This subsequently became a test for the presence of acids, the inference being that an acid has this capacity of changing the litmus to a red color from its original blue.

Professor Tyndall describes an experiment to prove that waves of ether issuing from a strong source, such as the sun or electric light, are competent to shake asunder the atoms of gaseous molecules, such as those of the sulphur and oxygen which constitute the molecule of sulphurous acid.

He enclosed the substance in a vessel, placing it in a dark room, and sending through it a powerful beam of light. At first nothing was seen; the vessel containing the gas seemed as empty as a vacuum. Soon, along the track of the beam, a beautiful sky-blue color was observed, due to the liberated particles of sulphur. For a time the blue grew more intense; it then became whitish; and from a whitish-blue it passed to a more or less perfect white. Continuing the action, the tube became filled with a dense cloud of sulphur particles which, by the application of proper means, could be rendered visible.[1] In this series of continuous changes, we find the one antecedent, giving initiative causal impulse, to be the beam of light. It was the one element introduced which started the several changes leading to the appearance of the sulphur visibly manifested. The one, therefore, is to be regarded as the cause of the other.

It is possible to represent this method by means of symbols in a manner similar to that of the method of agreement. Let $C$ be the supposed cause and $e$ the effect corresponding, while $S$ and $s$ denote the setting of antecedent and consequent respectively. We have, therefore, the following: —

$$S + C \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad s + e$$

Then, withdrawing $C$, we have the absence of $e$.

$$S \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad s$$

The inference then is that $C$ is the cause of $e$.

[1] Tyndall, *Use and Limit of the Imagination in Science*, p. 33.

In the method of agreement, a number of instances were taken agreeing only in the possession of two circumstances, — the cause and effect elements common to them all. In this method, only two instances are taken, and they must be precisely alike, with the one exception, — the presence of two circumstances in one, that is, the cause and the effect elements, and the absence of the same in the other. In the method of agreement, we compare the various phenomena, to note wherein they agree; in the method of difference, we compare the two phenomena, to note wherein they differ. The logical axiom underlying the two methods is substantially one and the same, differing only in its special adaptation in each case. The method rests on the assumption, which must be accepted as a fundamental postulate, that whatever can be eliminated from the various instances is not connected with the phenomenon under investigation in any causal manner; and the method of difference is based on the postulate that whatever cannot be eliminated is connected with the phenomenon by a causal law.

The method of difference is evidently the method by negation, which has already been indicated as the truly scientific process in induction. It is also pre-eminently the method of experiment rather than observation; for the withdrawal or introduction of forces can only be accomplished at will when we bring the phenomena under experimental control. At times, Nature herself may perform the experiment for us, and we stand simply as

observers to note the results. This is especially the case in the catastrophic phenomena, such as volcanic eruption, earthquakes, etc. Generally speaking, however, the method of difference is the process of man's manipulation to secure purposed results in which a causal relation is disclosed.

A question naturally suggests itself, What is there to determine the precise mode of experiment? We may have given a concrete whole of extreme complexity. In our experiment, which element shall we proceed to eliminate, in order to note the result? An answer may be given us through suggestions received from the results of the method of agreement which has been already applied to the problem. If it is not possible to avail one's self of this contribution from another sphere of investigation, then the complex whole must be broken up, as far as possible, into its simplest component parts, and one after another these parts, singly, then in pairs, and all other possible combinations, caused to be withdrawn, or their force neutralized, and the results in each case noted, as to whether the effect under investigation disappears. The exhaustion of all possible combinations must yield some definite result. Suppose, for instance, there is a complex antecedent involving four separable elements, as *A, B, C, D.* Withdraw severally *A, B, C,* and *D,* noting results; then withdraw, in turn, *AB, AC, AD, BC, BD, CD,* that is, the possible combinations of four elements taken two at a time; then withdraw *ABC,* then *BCD, ABD,* and *ACD,* that is, combinations of four elements

taken three at a time. By such a process there will be disclosed whether one element alone or whether a combination of two or more have produced the effect under investigation; also whether more than one element or combination of elements may have caused the effect.[1] The practical difficulty in separating the elements of a complex whole, and withdrawing the several combinations from the whole, renders this process in many cases impossible. The cause, however, is generally suspected. It may be suggested by the method of agreement, by analogy, or by that insight which at once declares certain combinations to be impossible and others irrelevant. There is generally a mental experiment in which the judgment rejects unlikely combinations, thus narrowing the field of investigation as preliminary to the experiments proper.

The method of difference is open to various criticisms; the most important are the following: —

1. In applying this method, we may be so easily misled, in supposing our two instances are precisely alike, with the one exception of the presence or absence of the supposed cause, and yet in reality the instances may differ radically, and yet we may be unable to detect this. A patient may have medicine administered to him, and begin at once rapidly to recover, and yet the very taking of the medicine in itself may have made such a mental impression, inducing confidence and hope, that the real cause of the recovery may be due wholly to

[1] This process has been illustrated and criticised at length in a striking manner by Venn, *Empirical Logic*, pp. 401 ff.

this mental reaction. Persons taking pills composed of inert substances have often given evidence of bodily effects wholly impossible to trace to the medicine itself. And yet this criticism is one of caution rather than of censure; for the defects are but difficulties which extreme care and insight may overcome.

2. It has been objected that this method may point out the cause in the concrete instance before the experimenter, but that this furnishes no basis whatsoever for a wider generalization that the effect in question is always produced by this cause. Sigwart has illustrated this objection by the instances in which typhus fever has been traced to the drinking of impure water.[1] The causal relation may be fully established in the cases investigated, but the universal proposition does not follow that wherever typhus fever appears, impure water has been drunk. This objection applies especially to cases of extreme complexity, where proximate causes alone can be discovered, and their ultimate nature, which may appear in various forms, is not revealed; for instance, the impure water is not in itself the ultimate cause of the typhus fever. It contains the poison germs, the real cause; they may be introduced into the system in some other way. Care, therefore, should be taken to reveal the cause in and by itself, and not the cause of the cause. The objection, therefore, may be in a measure overcome. To effect a generalization, moreover, of logical validity, it is necessary to supplement the method of

[1] Sigwart, *Logic*, Vol. II. p. 420.

difference by hypothesis and subsequent verifications, which will be described later on.

3. This method may lead to error in cases where the supposed causal element is regarded as the cause in its entirety, when it is in reality but a part of the cause. If one should plant seed in a garden and water only one half of the plot, and it should follow that only the watered part brought forth the leaf and flower, then an inference according to the method of difference might be drawn that the water caused the sprouting of the young plants. And yet it must be regarded simply as contributory to the real cause. Such a difficulty may be obviated by a careful discrimination in the analysis of the phenomenon investigated.

4. Sometimes a liberating cause may be revealed by a strict interpretation of the method of difference, when the real cause is more obscure, and may be overlooked. A stone may strike a can of dynamite and the explosion which occurs may be traced to the impact of the stone. It is the one element of difference introduced in the sphere of the observed phenomena, with the consequent result. The force existing as a potential is naturally obscure, and apt to elude observation. Therefore, whenever a cause disclosed by the method of difference seems to be out of all proportion to the effect, it at once suggests the probability that a potential force not discerned by our powers of observation has been the real cause, and the former a conditioning cause merely. Another illustration of this is the experiment of Priestley which led to

his discovery of oxygen in 1774. He placed some oxide of mercury upon the top of quicksilver in an inverted glass tube filled with that metal and standing in mercury; he then heated the oxide by means of a glass lens and the sun's rays, and obtained a gas, which he called "nitrous air," afterwards designated as oxygen. The heat in this case was the sole element of difference between the two instances, one in which there was no gas, and the second after application of the heat, when the gas was present. Here the heat must be regarded as the liberating and not in any sense the producing cause. Again, as Lotze says, "the fact that with the destruction of a single part of the brain a definite psychical function ceases, is no proof that just this single part was the organ which alone produced that function." [1]

In addition to the difficulties attending this method which have been enumerated and which have to do with the logical theory of the method, there are also difficulties of a practical nature which arise in the actual application of this method in experimental inquiry. They are as follows: —

1. Care must be taken that, in the two phenomena compared, with and without the supposed cause, there shall not be an interval of time elapsing, in which period some other cause might be introduced unknown to the investigator, and yet capable of producing the result, or else of neutralizing some force that is present and itself capable of producing the result. For instance, if a chemical

[1] Lotze, *Logic*, p. 322.

compound be left for an appreciable time, we may notice certain changes and be able to assert positively that no new element has been introduced, and yet the action of the air may in itself have been sufficient to work these changes. When the two phenomena to be compared can be presented for inspection simultaneously, this difficulty is obviated. This is illustrated in an experiment devised to exhibit the presence of light effects in the spectrum beyond the violet rays; that is, beyond the place where the spectrum seems to end. A sheet of paper is taken, the lower part of which is moistened with a solution of sulphate of quinine, while the upper part remains dry. Let the image of the solar ray fall upon this sheet; the spectrum preserves at the top of the sheet in the dry portion of the paper its ordinary appearance, while in the moistened portion a brilliant phosphorescence appears beyond the region of the violet rays. Here the dry and wet portions are simultaneously presented, and there is but one point of difference between the two. The inference, therefore, is readily drawn that the solution of sulphate of quinine is a substance sensitive to the ultra-violet portion of the sun's rays, the phosphorescence being the effect of these rays upon the solution.

2. Extreme care must be taken that, in the withdrawing of any element in the course of the experiment, no other element is inadvertently introduced, and that, in adding any element, no existing element or combination of elements is destroyed, or their effect neutralized. Mr. Venn has admirably illus-

trated this difficulty, and I give the following quotation in full from him: "We suppose that when we have put a weight into one pan of a pair of scales we have done nothing more than this, or can at any rate by due caution succeed in doing nothing more. But if we exact the utmost rigidity of conditions, we easily see that we have done a great deal more. Our bodies are heavy, and therefore the mere approach to the machine has altered the magnitude and direction of the resultant attraction upon the scales. Our bodies are presumably warmer than the surrounding air; accordingly, we warm and therefore lighten the air in which the scales hang, and if the two scales and their contents are not of the same volume, we at once alter their weight as measured in the air. Our breath produces disturbing currents of air. Our approach affects the surface of the non-rigid floor or ground on which the scales stand, and produces another source of disturbance, and so on through the whole range of the physical forces." [1]

In the Report of the British Association, 1881, an account is given of Professor G. H. Darwin's experiments to measure the lunar disturbance of gravity at the Cavendish Laboratory by means of an extremely delicate pendulum. It was found that approaching the pendulum in order to observe its reading, the surface level of the stone floor on which the instrument stood was deflected by the weight of the observer. As he stood to take the reading, the shifting of his weight from one leg to

[1] Venn, *Empirical Logic*, p. 416.

the other was perceptible; so it became necessary to observe the reading by a telescope from a distance, or to adopt some similar plan.[1]

Faraday was able at will to produce or remove a magnetic force, through the revealed properties of the electromagnet. Many of his experiments would have been impossible if it had been necessary to remove a cumbersome magnet and reinstate it again and again in his experiments. The electromagnet, however, could produce or destroy the presence of magnetic force without any incidental perturbations. Thus Faraday was enabled to prove the rotation of circularly polarized light by the fact that certain light ceased to be visible when the electric current of the magnet was cut off, and instantly reappeared when the current was re-established. Faraday says of the experiment himself: "These phenomena could be reversed at pleasure, and at any instant of time, and upon any occasion, showing a perfect dependence of cause and effect."[2]

3. In some cases it is impossible to remove an element which is supposed to be the cause of an effect under investigation. Its removal might cause the destruction or the impairing of the whole phenomenon. The force, therefore, that cannot be eliminated must be neutralized by an equal and opposing force. For instance, the force of gravity cannot be eliminated; it must, therefore, be counterbalanced by some device of the investigator.

[1] Quoted by Venn in *Empirical Logic*, p. 419.

[2] *Experimental Researches in Electricity*, Vol. III. p. 4.

In chemistry the removal of an element from a compound may be impossible without destroying utterly the compound itself; in such a case, also, a neutralizing agent must be introduced. Darwin wished to prove that the odor of flowers is attractive to insects irrespective of the attraction of color. He therefore covered certain flowers with a muslin net, which still attracted the insects to them.[1]

The following illustrations may serve further to exhibit the various features of the method of difference: —

Mr. Robert Mallet gives the following interesting account of his visit to Faraday: "It must be now eighteen years ago when I paid him a visit, and brought some slips of flexible and *tough* Muntz's yellow metal, to show him the instantaneous change to complete brittleness with rigidity produced by dipping into pernitrate of mercury solution. He got the solution and I showed him the facts; he obviously did not doubt what he saw me do before and close to him; but a sort of experimental instinct seemed to require he should try it himself. So he took one of the slips, bent it forward and backward, dipped it, and broke it up into short bits between his own fingers. He had not before spoken. Then he said, 'Yes, it *is* pliable, and it *does* become instantly brittle.'"[2] Here the experiment with and without the significant antecedent and consequent

[1] Darwin, *Cross and Self Fertilization*, p. 374.

[2] Gladstone, *Michael Faraday*, p. 175.

result indicates the causal relation, especially as the instantaneous effect precludes the possibility of the operation of any other cause.

Another experiment of Faraday's is that of his investigation of the behavior of Lycopodium powder on a vibrating plate. It had been observed that the minute particles of the powder collected together at the points of greatest motion, whereas sand and all heavy particles collected at the nodes, where the motion was least. It occurred to Faraday to try the experiment in the exhausted receiver of an air pump, and it was then found that the light powder behaved exactly like heavy powder. The inference was that the presence of air was the condition of importance, because it was thrown into eddies by the motion of the plate, and carried the Lycopodium powder to the points of greatest agitation. Sand was too heavy to be carried by the air.[1]

Sir John Lubbock gives an account of experiments performed upon insects to prove that the sense of smell is in some way connected with their antennæ. One experiment was performed by Forel, who removed the wings from some bluebottle flies and placed them near a decaying mole. They immediately walked to it, and began licking it and laying eggs. He then took them away, and removed the antennæ, all other circumstances remaining the same as before, after which, even when placed close to the mole, they did not appear to perceive it. Another experiment similar

[1] Jevons, *Principles of Science*, p. 419.

to this was tried by Plateau, who put some food of which cockroaches are fond on a table and surrounded it with a low circular wall of cardboard. He then put some cockroaches on the table; they evidently scented the food, and made straight for it. He then removed their antennæ, after which, as long as they could not see the food, they failed to find it, even though they wandered about quite close to it.[1]

Another experiment is that of Graber to prove the sense of hearing in insects. He placed some water-boatmen (*Corixa*) in a deep jar full of water, at the bottom of which was a layer of mud. He dropped a stone on the mud, but the beetles, which were reposing quietly on some weeds, took no notice. He then put a piece of glass on the mud, and dropped a stone on to it, thus making a noise, though the disturbance of the water was the same as when the stone was dropped on the mud. The water-boatmen, however, then at once took flight.[2]

An illustration of the method of difference occurs in the so-called *blind experiments*, which are often made in chemistry especially. As Professor Jevons has described such an experiment: "Suppose, for instance, a chemist places a certain suspected substance in Marsh's test apparatus and finds that it gives a small deposit of metallic arsenic, he cannot be sure that the arsenic really proceeds from the suspected substance; the impurity of the zinc or

[1] Lubbock, *On the Senses, Instincts, and Intelligence of Animals*, p. 45.

[2] Lubbock, *On Senses, etc.*, p. 75.

sulphuric acid may have been the cause of its appearance. It is therefore the practice of chemists to make what they call blind experiments, that is, to try whether arsenic appears in the absence of the suspected substance. The same precaution ought to be taken in all important analytical operations. Indeed it is not merely a precaution, it is an essential part of any experiment. If the blind trial be not made, the chemist merely assumes that he knows what would happen."[1]

[1] Jevons, *Principles of Science*, p. 433.

# CHAPTER IX

## The Joint Method of Agreement and Difference

It has already been shown that the method of difference is sometimes not available, inasmuch as it may be neither possible nor practicable to remove from the phenomenon to be investigated the suspected causal element without destroying the phenomenon itself. Sometimes, too, it is impossible even to neutralize the effect of the causal element if it is allowed to remain as an integral part of the phenomenon. This is especially the case in all vital phenomena, and also in many chemical phenomena. Therefore another method is resorted to, which is known as the joint method of agreement and difference. Inasmuch as the suspected causal element cannot be removed, we must select another phenomenon as much like the former as possible, which is, however, characterized by the absence of the causal element. By the simple method of difference, two instances only need be compared, the one with and the other without the causal element, but agreeing precisely in every other particular. In the joint method, the instances with and without

the causal element, differ it may be in several particulars. A number of varying instances must therefore be selected so as to eliminate the possibility of any of these differing characteristics being the cause in question. Therefore two sets of instances are collected, and compared. The one set comprises all the positive instances having the presence of the supposed causal element, and the second set consists of the negative instances having the supposed causal element absent altogether. The validity of the method depends upon the similarity of the two sets of instances. As the similarity increases, the method approximates to the simple method of difference.

*The Canon of the Joint Method.* — If several instances in which the phenomenon occurs have only one circumstance in common, while several instances in which it does not occur have nothing in common save the absence of that circumstance; the circumstance in which alone the two sets of instances differ, is the effect, or cause, or a necessary part of the cause, of the phenomenon.

The symbolical representation of this method may be exhibited as follows, using a similar notation to that employed in the previous methods: —

I. Table of positive instances.

$$
\begin{array}{lcl}
S + C & \ldots\ldots\ldots & s + e \\
S' + C & \ldots\ldots\ldots & s' + e \\
S'' + C & \ldots\ldots\ldots & s'' + e \\
S''' + C & \ldots\ldots\ldots & s''' + e \\
\text{etc.} & & \text{etc.}
\end{array}
$$

II. Table of negative instances.

| | |
|---|---|
| $S_{\prime}$ . . . . . . . . . . . . | $s_{\prime}$ |
| $S_{\prime\prime}$ . . . . . . . . . . . . | $s_{\prime\prime}$ |
| $S_{\prime\prime\prime}$ . . . . . . . . . . . . | $s_{\prime\prime\prime}$ |
| etc. | etc. |

In the two sets of instances, the following conditions must be observed in order to render the method valid: —

1. $S + C$, $S' + C$, $S'' + C$, $S''' + C$, etc.,

must be so varied that they reveal but one constant element, common to them all, as $C$. It may be that $S$ will resemble $S'$ in more marks than the one, namely $C$, and this may be true of any two or more instances; however, taken all together, they must possess but the one common element, $C$.

2. In the same way $S_{\prime}$ may resemble $S_{\prime\prime}$ in more marks than merely the absence of $C$ and so for any two or more instances in the series $S_{\prime}$, $S_{\prime\prime}$, $S_{\prime\prime\prime}$, etc. However, the one characteristic common to them all must be the absence of $C$.

3. If in the instances chosen an element is common to all in addition to $C$, or in the second set its absence, then additional instances must be added to the tables both positive and negative in order to secure this all-important condition of elimination through suitable variation.

4. Moreover, the two series, positive and negative, must have their settings similar. $S_{\prime}$, $S_{\prime\prime}$, $S_{\prime\prime\prime}$, etc., must resemble $S'$, $S''$, $S'''$, etc.; otherwise the negative instances would not be significant. They must

be chosen from the same sphere as the positive, that they may be similar. It is possible to multiply negative instances *ad infinitum*, which, however, would furnish no ground for any inference, because they would be wholly irrelevant to the problem under investigation.

5. If $S_{\prime}$ is so similar to $S'$ as to be identical with it, and also $s_{\prime}$ pass over into $s'$; then we have the method of difference in its pure form: —

$$S' + C \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad s' + e$$
$$S' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad s'$$

Here the setting, instead of being similar in the two cases, is the same in each.

The following is an experiment of Sir John Lubbock's concerning the sense of smell in insects, which I have chosen as illustrating this method of inductive research. He took a large ant and tethered her on a board by a thread. When she was quite still, he brought a tuning-fork into close proximity to her antennæ, but she was not disturbed in the least. He then approached the feather of a pen very quietly, so as almost to touch first one and then the other of the antennæ, which, however, did not move. He then dipped the pen in the essence of musk and did the same; the antenna was slowly retracted and drawn quite back. He then repeated the same with the other antenna, and with like result. Care was taken throughout not to touch the antennæ. Lubbock then repeated the experiment with a number of different ants, and using various substances. The results in all cases were

the same, and the inference was naturally drawn that the antennæ possessed the sense of smell. In these experiments, various substances were taken having nothing in common save the odor of musk that had been placed upon them.

In some cases it is not possible to discover positive instances in which the only common element is the suspected cause. In such cases the method is not conclusive in its results, although it may attain a high degree of probability, if all the common elements save the suspected cause-element are known to be irrelevant, or can in any other way be proved to have no influence whatsoever upon the result. For instance, an illustration is often given of this method, which fails in the manner just described. A man is attempting to discover whether a particular article of food disagrees with him. He notices several occasions, a large number if you please, when he has eaten this particular kind of food, and has soon after experienced distress. These are the positive instances. This peculiar distress has never been experienced when he has abstained from the food in question. The inference is that this food has caused the distress. In the various instances, however, the sole element in common is not merely the taking or not taking the food. The person's whole bodily organism is common to all the instances. Within it, unforeseen complications independent of this article of food might have caused the trouble. In such cases, number of instances must be resorted to in order to render the possibility of a coincidence impossible.

So also in such cases as the treatment of any given disease in a hospital. An experiment may be tried in the treatment, say, of typhoid fever. One ward may be subjected to a particular kind of treatment, and another ward not subjected to that treatment. If recovery is hastened in the one and retarded in the other case, an inference may be drawn as to efficacy of this treatment. In these instances again, while they are all different patients, still the nursing, surroundings, etc., are common to them all. It must be shown that these are present both in the negative and positive instances, and equally capable of accomplishing the effect if they had been real causes. They may therefore be eliminated in comparing the two sets of instances, because common both to the negative and positive cases. In this also resort must be had to the number of instances in order to eliminate chance coincidences. The presence of common elements in excess of the common causal element may be represented according to the symbolical notation of the joint method, by the introduction of another symbol $x$. Let $x$ stand for that which is common to all instances in addition to the common element $C$. We then have:—

I. Set of positive instances.

$$\begin{array}{lcl} S + C + x & \ldots\ldots\ldots & s + e \\ S' + C + x & \ldots\ldots\ldots & s' + e \\ S'' + C + x & \ldots\ldots\ldots & s' + e \\ S''' + C + x & \ldots\ldots\ldots & s''' + e \\ \text{etc.} & & \text{etc.} \end{array}$$

II. Set of negative instances.

$S_{\prime} + x$ . . . . . . . . . . . . $s_{\prime}$
$S_{\prime\prime} + x$ . . . . . . . . . . . . $s_{\prime\prime}$
$S_{\prime\prime\prime} + x$ . . . . . . . . . . . . $s_{\prime\prime\prime}$
etc. etc.

We observe $x$ in all instances both positive and negative. Being present when the effect occurs and when it does not, indifferently, we can at once infer that $x$ is not the whole cause of $e$. However, it may have united with $C$ in the first set of instances to produce the effect $e$, so that $C$ without $x$, or some part or parts of $x$, could not alone produce the effect $e$. In all such cases the exact force of $x$ must be estimated in some other way. If $x$ is extremely complex, or subject to change from forces emanating from within itself, as in the case of organic phenomena, then it becomes extremely difficult to determine $x$; and consequently the method of agreement and difference does not yield as exact results. As long as the force of $x$ remains unknown, it becomes the source of possible disturbance, which may wholly vitiate the results attained.

Mr. Darwin, in his experiments upon cross and self fertilization in the vegetable kingdom, placed a net about one hundred flower heads, thus protecting them from the bees and from any chance of fertilization by means of the pollen conveyed to them by the bees. He at the same time placed one hundred other flower heads of the same variety of plant where they would be exposed to the bees, and, as he observed, were repeatedly visited by them.

Here we have the two sets of instances, in one the flowers accessible to the bees, and in the other, not accessible. He obtained the following result. The protected flowers failed to yield a single seed. The others produced 68 grains weight of seed, which he estimated as numbering 2720 seeds. Cross-fertilization as the cause in this case is thus proved. The common element in all these instances, however, is not merely the presence in one case and the absence in the other of the bees; there is also the element of the common plant structure running through all of the two hundred instances. This element is, however, of such an unvarying nature in all the instances, and the number observed so many as to eliminate the possibility of any given plant structure possessing unobserved peculiarities sufficient to produce the result in question. It may therefore be considered as an inert element as regards the effects noticed in the one and absent in the other set of instances.

Sir John Lubbock, in his researches concerning the different functions of the two kinds of eyes in insects, illustrates the joint method in its general features. The two kinds of eyes are the large compound eyes, situated one on each side of the head, and the ocelli, or small eyes, of which there are generally three, arranged in a triangle between the other two. He wished to determine the precise function of the small eyes, the ocelli; and he has gathered together the following facts. Plateau has shown that caterpillars, which possess ocelli, but no compound eyes, are very short-sighted, not seeing above one to two centimetres. He has also proved

by experiments that spiders, which have ocelli but no compound eyes, are very short-sighted; they were easily deceived by artificial flies of most inartistic construction, and even hunting spiders could not see beyond ten centimetres (four inches). Lubbock experimented on this point with a female spider, which, after laying her eggs, had rolled them into a ball, and had enveloped the whole with a silken bag which she carried about with her. Having captured the female and having taken the bag of eggs from her, he placed it on a table about two inches in front of her. She evidently did not see it. He then pushed it gradually towards her, but she took no notice till it nearly touched her, when she eagerly seized it. He then took it away a second time, and put it in the middle of the table, which was two feet four inches by one foot four, and had nothing else on it. The spider wandered about for an hour and fifty minutes before she found it, apparently by accident. He then took it away again and put it down as before, when she wandered about for an hour without finding it. Like experiments were tried with other spiders and with the same results. Plateau also experimented with scorpions which had ocelli and no compound eyes. They appeared scarcely to see beyond their own pincers. Moreover, the ocelli are especially developed in insects, such as ants, bees, and wasps, which live partly in the open light and partly in the dark recesses of nests. Again, the night-flying moths all possess ocelli. On the other hand, however, they are entirely absent in all butterflies, with, according

to Scudder, but one exception, namely, the genus Pamphila. Forel, moreover, varnished the compound eyes of various insects which had ocelli as well. The latter, however, he allowed to remain in their natural state. Inasmuch as their habits of flight required powers of vision in these insects extending to a considerable distance, it happened that when placed on the ground they made no attempt to rise; while, if thrown into the air, they flew first in one direction and then in another, striking against any object that came in their way, and being apparently quite unable to guide themselves. They flew repeatedly against a wall, falling to the ground, and unable to alight against it, as they did so cleverly when they had their compound eyes to guide them. All these instances, taken together in their positive and negative aspects, led Sir John Lubbock to infer that the ocelli were useful in dark places and for near vision, while the compound eyes were for the light and more distant vision.[1]

Another illustration of this method may be found in Darwin's account of the extreme tameness of the birds in the Galapagos and Falkland islands. I quote some extracts from his narrative, in which it will be seen that Darwin's inferences follow from his comparison of the positive and negative instances before him. He says: "This tameness of disposition is common to all the terrestrial species of these islands in the Galapagos Archipelago, namely,

[1] Lubbock, *On the Senses, Instinct, and Intelligence of Animals*, pp. 175 ff.

to the mocking-thrushes, the finches, wrens, tyrant flycatchers, the dove, and carrion-buzzard. All of them often approached sufficiently near to be killed with a switch, and sometimes, as I myself tried, with a cap or hat. A gun is here almost superfluous; for, with the muzzle, I pushed a hawk off the branch of a tree. In Charles Island, which had been colonized about six years, I saw a boy sitting by a well with a switch in his hand, with which he killed the doves and finches as they came to drink. He had already procured a little heap of them for his dinner; and he said that he had constantly been in the habit of waiting by this well for the same purpose. The Falkland Islands offer instances of birds with a similar disposition. The snipe, upland and lowland goose, thrush bunting, and even some true hawks, are more or less tame. The black-necked swan is here wild, and it was impossible to kill it. It, however, is a bird of passage, which probably brought with it the wisdom learned in foreign countries.

From these several facts, we may, I think, conclude that the wildness of birds with regard to man, is a particular instinct directed against *him* and not dependent on any general degree of caution arising from other sources of danger; secondly, that it is not acquired by individual birds in a short time, even when much persecuted, but that in the course of successive generations it becomes hereditary. With domesticated animals we are accustomed to see new mental habits or instincts acquired and rendered hereditary, but with animals

in a state of nature it must always be most difficult to discover instances of acquired hereditary knowledge. In regard to the wildness of birds towards man, there is no way of accounting for it except as an inherited habit: comparatively few young birds, in any one year, have been injured by man in England, yet almost all, even nestlings, are afraid of him; many individuals, on the other hand, both at Galapagos and at the Falklands, have been pursued and injured by him, but yet have not learned a salutary dread of him."[1]

I have given this quotation somewhat at length in order to show the method of a great investigator in the realm of nature; and that it may be seen how naturally he falls into the method of comparing positive and negative sets of instances relative to the object of research. The animal and vegetable kingdoms are especially adapted to the application of this joint method, and therefore it is in biology that it is most frequently employed and where it has yielded the most fertile results.

The advantage of the joint method over the simple method of agreement is that it largely eliminates the possibility of there being any other cause of the given phenomenon than the one disclosed by the operation of this method. The method of agreement, as we have seen, often fails of a definite result owing to the plurality of causes. The joint method tends to indicate the one and only cause, and when the instances are rigorously selected according to the conditions of the canon, there is a

[1] Darwin, *Voyage of a Naturalist*, Vol. II. pp. 172 f.

high degree of probability that the sole cause is discovered. Mr. Mill at this point claims too much for the method in insisting that it gives a certainty regarding the sole cause, when the requirements are perfectly realized. It is impossible to realize the requirements perfectly. In selecting the negative instances, we are never sure that we have compassed the entire sphere of *significant* negative instances. We may, however, attain results highly probable in this regard, though they may not reach an absolute certainty. Such a statement is more moderate in its expression, and practically it assures as satisfactory results.

# CHAPTER X

## The Method of Concomitant Variations

The method of concomitant variations is a process of determining a causal relation when, as an element in an antecedent varies in intensity, greater or less, there is observed a corresponding or concomitant variation in the consequent.

*Canon of the Method of Concomitant Variations.*— Whatever phenomenon varies in any manner, whenever another phenomenon varies in some particular, is either a cause or an effect of that phenomenon, or is connected with it through some fact of causation.

The latter clause of this canon provides for that circumstance in which the varying elements may both be concomitant effects of a common cause. When we are assured of the absence of any possible common cause to which we can assign the two phenomena as effects, then they must be related between themselves as cause and effect. A simple illustration of this method is the rise of the mercury in the thermometer owing to the increase of heat; its fall, whenever there is decrease of heat. One varies as the other concomitantly, and we infer a causal relation that we at once proceed to generalize without hesitation.

The symbolical representation of this method is as follows:—

$$\begin{array}{lcl} S + C & \ldots\ldots\ldots & s + e \\ S + C \pm dC & \ldots\ldots & s + e \pm de \\ \text{etc.} & & \text{etc.} \end{array}$$

Then $C$ is the cause of $e$.

I have used $dC$, and $de$ to denote the increments or decrements of the cause and effect respectively. This method is used generally when the method of difference is impossible, owing to the fact that the supposed causal element cannot be made to vanish wholly. In all such cases a variation of the element is resorted to, and the corresponding result observed. Heat is relative and not absolute, as also the height of mercury in the tube. The relation is determined, therefore, by variations, greater and less. This method is also used to supplement the results of other methods by which a causal relation has been determined, but not in exact quantitative terms. It may be known that a certain phenomenon $C$ is always the cause of a certain effect $e$, and the method of concomitant variations will then be of use in determining precisely how much of a variation in $C$ will cause a specified variation in $e$. A law finds scientific expression only when stated in terms of exact quantitative relation between variations in antecedent and consequent. We may express the law of universal attraction in a vague way that bodies always attract each other and the greater attraction when the bodies are nearer together, and the larger they are. But this statement needs to

be recast in terms exhibiting the precise quantitative variation. Bodies attract each other directly as the product of their masses, and inversely as the square of their distance. It is evident that the special function of this method of concomitant variations consists in just this quantitative determination. In one respect, therefore, it may be regarded as a substitute for the method of difference, and in another way as a supplement to the method of difference in leading to quantitatively determinate results.

The quantitative variation between antecedent and consequent may be either direct or inverse variation. The former is when one increases as the other increases, or when one decreases as the other decreases. The inverse is when one decreases as the other increases, or *vice versa.* This may be expressed symbolically

$$S + C \pm dC \quad . \quad . \quad . \quad . \quad s + e \mp de$$

We have, for instance, Boyle's law as regards the variation of volume of gases according to the pressure; that is, when we double the pressure, we halve the volume. This may be proved experimentally. The method also was used by Ricardo to prove his law that the rate of profits varies in inverse ratio to the rate of wages. We have also the tendency observed in respect to increase of crimes, when there is decrease of opportunities for labor.

The expression of a law in terms of the quantitative relation between antecedent and consequent may be facilitated by a graphic representation of the

same, through corresponding abscissæ and ordinates. The varying antecedents, for instance, may be laid off on the axis of $X$, and each several consequent represented by the corresponding ordinates. The resulting curve thus obtained will represent the law of their mutual relation. If the equation of the curve can be determined, it will represent the mathematically exact expression of the law in question. If this is not possible, it may prove at least suggestive of the law which otherwise might have remained concealed. This graphical method is especially useful in dealing with physical phenomena. "If the abscissæ represent intervals of time, and the ordinates corresponding height of the barometer, we may construct curves which show at a glance the dependence of barometric pressure upon the time of day. Such curves may be accurately drawn by photographic processes on a sheet of sensitive paper placed behind the mercurial column, and made to move past it with a uniform horizontal velocity by clockwork. A similar process is applied to the temperature and electricity of the atmosphere, and to the components of terrestrial magnetism."[1]

This method, moreover, has the advantage of the psychological impression which it makes. The mind is more susceptible to the perception of variation in forces where the change is apparent to the senses, than to the perception of a constant force, whose constant character thereby conceals its nature and function from the senses. Synchronous

[1] Thomson and Tait, *Elements of Natural Philosophy*, Vol. I. p. 119.

changes attract the attention, and admit of ready comparison, as we follow out the variations from point to point. We may ring a bell in a vacuum, and detect no sound whatsoever, and then allow the air to enter gradually. We notice that as the air enters more and more freely, the sound grows louder and louder. The relation of cause and effect is thus demonstrated to the senses in the most vivid manner possible. The variations are exhibited side by side, and thus, presented together in their concomitant relation, produce the deeper impression.

This method is of special advantage in all experiments where the intensity of the forces can be varied at will and the consequent effects exhibited in some palpable manner. The determination of the heat rays in the solar spectrum is accomplished by this method. The spectrum may be received upon a plate pierced with a narrow slit, through which the rays can act upon a thermo-electric pile, which will indicate by deflections of a needle the varying intensity of the heat in the several rays of the spectrum. Now, move the slit through the whole extent of the spectrum, beginning with the violet portion. While in the violet, the indigo, the blue, and even the green, the needle of the thermoscopic apparatus will be deflected but slightly, it will indicate an amount of heat increasing as the slit crosses the yellow, next the orange, then the red; and then beyond the red, and entering the dark portion of the spectrum, we find the greatest deflection of all. The maximum of heat is therefore in a region beyond the observation of the

senses when unaided by experimental device; and yet revealed conclusively by this method.[1]

Professor Tyndall performed a very interesting experiment to prove that the cloud of darkness surrounding flames of great heat was due to the fact that the heat consumed the floating motes in the air which serve to scatter the light which is visible only when thus diffused. The phenomenon which he endeavored to explain was somewhat as follows: Beneath a beam of electric light, a red-hot poker was placed, and from it black wreaths as of smoke were seen to ascend. A large hydrogen flame being employed, it produced whirling masses of darkness far more copiously than the poker. Of this Professor Tyndall remarked: "Smoke was out of the question; what then was the blackness? It was simply that of stellar space; that is to say, blackness resulting from the absence from the track of the beam of all matter competent to scatter its light. When the flame was placed below the beam, the floating matter was destroyed *in situ;* and the air freed from this matter rose into the beam, jostled aside the illuminated particles, and substituted for their light the darkness due to its own perfect transparency. Nothing could more forcibly illustrate the invisibility of the agent which renders all things visible. The beam crossed, unseen, the black chasm formed by the transparent air, while at both sides of the gap the thick-strewn particles shone out like a luminous solid under the powerful illumination."[2]

[1] Saigey, *The Unity of Natural Phenomena*, p. 61.
[2] Tyndall, *Fragments of Science*, p. 280.

Such being the phenomenon and Professor Tyndall's explanation, it will be seen that he proceeded according to the method of concomitant variations in the following experiment of many which he performed to substantiate this theory: —

A platinum tube, with its plug of platinum gauze, was connected with an experimental tube, through which a powerful beam could be sent from an electric lamp placed at its end. The platinum tube was heated till it glowed feebly but distinctly in the dark. The experimental tube was then exhausted, and filled with air that had passed through the red-hot tube. A considerable amount of floating matter which had escaped combustion was revealed by the electric beam.

Then the tube was raised to a brighter redness and the air permitted to pass slowly through it. Though diminished in quantity, a certain amount of floating matter passed into the exhausted experimental tube.

The platinum tube was rendered still hotter; a barely perceptible trace of the floating matter now passed through it. The experiment was repeated, with the difference that the air was sent more slowly through the red-hot tube. The floating matter was totally destroyed.

The platinum tube was now lowered until it bordered upon a visible red heat. The air, sent through it still more slowly than in the last experiment, carried with it a cloud of floating matter. Professor Tyndall's commentary upon this experiment is as follows: "If, then, the sus-

pended matter is destroyed by a bright red heat, much more is it destroyed by a flame, whose temperature is vastly higher than any employed in this experiment. So that the blackness introduced into a luminous beam where a flame is placed beneath it is due, as stated, to the destruction of the suspended matter."[1]

Professor Tyndall also supplemented this experiment by one which was according to the joint method of agreement and difference. He prepared oxygen so as to exclude all floating particles, and found that when blown into the beam, darkness was produced; also that hydrogen, nitrogen, carbonic acid, and coal-gas, when prepared in a similar way, each produce darkness when poured or blown into the beam. These instances, combined with various positive instances of illumination of mote-strewn currents of air, illustrate the method of agreement and difference.

An additional experiment, according to the method of difference, was as follows: Professor Tyndall placed an ordinary glass shade in the air with its mouth downward. This permitted the track of the beam to be seen crossing it. Letting coal-gas, or hydrogen, enter the shade by a tube reaching to its top, the gas gradually filled the shade from the top downward. As soon as it occupied the space crossed by the beam, the luminous track was instantly abolished. Lifting the shade so as to bring the common boundary of gas and air above the beam, the track flashed forth. After the shade was

[1] Tyndall, *Fragments of Science*, pp. 283, 284.

full, he inverted it; thereupon the gas passed upward like a black smoke among the illuminated particles.[1]

The method of concomitant variations is not only capable of illustration by laboratory methods and devices; it finds abundant illustration as well in the realm of nature, where observation alone becomes the instrument of investigation and where experiment is impossible or impracticable. Lyell, in his *Principles of Geology*, gives a very interesting account of the alternate elevation and subsidence of the temple of Jupiter Serapis, at Pozzuoli, on the Bay of Naples.[2] It is situated in proximity to several volcanoes, Vesuvius, however, being at some distance. It has been observed that there is a certain connection between each era of upheaval, and a local development of volcanic heat; and on the other hand, between each era of depression, and the local quiescent condition of volcanic phenomena. Before the Christian era, when Ischia was in a state of eruption, and Avernus and other points in the Phlegræan fields were celebrated for their volcanic character, it was observed that at that time the ground on which the temple stood was several feet above water. Vesuvius was then regarded as a spent volcano. After the Christian era, Vesuvius became active and then scarcely a single eruption occurred in Ischia or around the Bay of Baiæ. It was observed that at that time the temple was sinking. Vesuvius then became quiet for five centuries pre-

[1] Tyndall, *Fragments of Science*, pp. 284, 285.
[2] Chapter XXX.

ceding the eruption of 1631, and during that period the Solfatara was in eruption in 1198, Ischia in 1302, and Monte Nuovo was formed in 1538. Then the foundations of the temple were observed to be rising again. Vesuvius became active after that, and has continued so ever since, and during this time the temple has been subsiding. The inference is that as the subterranean heat increases, and lava forming without obtaining an easy vent like that afforded by Vesuvius, the surface is elevated, but when the rocks below are cooling and contracting, the pent-up fire being withdrawn in the eruption of the great Vesuvius, then there is a corresponding subsidence.

The observation of concomitant variations is furthermore illustrated in Darwin's researches concerning the formation of coral reefs, as regards the question which some naturalists have raised as to which part of the coral reef is most favorable to the growth of coral.[1] He adduces the following facts, most of which are the direct result of his observations: "The great mounds of living Porites and of Millepora round Keeling atoll occur exclusively on the extreme verge of the reef, which is washed by a constant succession of breakers. At the Marshall Islands the larger kinds of coral which form rocks measuring several fathoms in thickness prefer the most violent surf. The outer margin of the Maldiva atolls consists of living corals, and here the surf is so tremendous that even large ships have been thrown, by a single heave of the sea,

[1] Darwin, *Coral Reefs*, pp. 87 f.

high and dry on the reef, all on board thus escaping with their lives. In the Red Sea the strongest corals live on the outer reefs and appear to love the surf. From these facts it is certain that the strongest and most massive corals flourish where most exposed. The less perfect state of the reef of most atolls on the leeward and less exposed side, compared with its state to the windward, and the analogous case of the greater number of breaches on the rear sides of those atolls in the Maldiva Archipelago, which afford some protection to each other, are obviously explained by this circumstance." There seems to be here a combination of the method of agreement with that of concomitant variations. And such a combination may be employed to advantage in cases where the phenomena under investigation show forces under varying degrees of intensity; the causal relation is more apparent, and the possibility of fortuitous coincidence is largely eliminated if a number of instances can be collected in which the forces manifest themselves in varying degrees. Accumulation of instances, showing concomitant variations in the forces observed, corresponds to the actual variations which in an experiment are effected by the investigator himself. In such observed instances, however, we cannot always have before us the variations expressed continuously. There are evident gaps that must be interpolated mentally. In the experiment, however, of whatever nature, the degrees of intensity can be exhibited continuously, one degree merging into another through inappreciable increments. There is thus

a gradation which has no gaps to be filled, and the psychological impression is thereby heightened.

By the method of concomitant variations it is possible also to represent to the mind the magnitude of an unknown force, or unobservable force by comparison with the intensity of a known force, which lies within the sphere of observation. For instance, Mr. Darwin gives an interesting account in his narrative of the finding near the shores of the Plata a group of vitrified silicious tubes which had been formed by lightning entering loose sand. The internal surface of such tubes is completely vitrified, glossy, and smooth, and the tubes themselves are generally compressed, and have deep longitudinal furrows so as closely to resemble a shrivelled vegetable stalk, or the bark of an elm or cork tree. Their circumference is about two inches, but in some fragments which are cylindrical and without any furrows, it is as much as four inches. Judging from the uncompressed fragments, the measure or bore of the lightning proved to be about one inch and a quarter. In contrast with the force of lightning as thus revealed in its effects, Mr. Darwin cites some experiments performed in Paris by an artificial force of great magnitude indeed and yet with results that seem insignificantly small in comparison. He says: "At Paris, M. Hatchette and M. Beudant succeeded in making tubes in most respects similar to these fulgurites by passing very strong shocks of galvanism through finely powdered glass: they failed, however, both with powdered felspar and quartz. One tube, formed with pounded glass,

was very near an inch long, namely, .982, and had an internal diameter of .019 of an inch. When we hear that the strongest battery in Paris was used, and that its power on a substance of such easy fusibility as glass was to form tubes so diminutive, we must feel greatly astonished at the force of a shock of lightning, which, striking the sand in several places, has formed cylinders in one instance at least thirty feet long, and having an internal bore, where not compressed, of full an inch and a half; and this in a material so extraordinarily refractory as quartz!"[1]

The method of concomitant variations may be used in regard to phenomena whose nature is such as seemingly to indicate a constant law of variation, and yet inferences based thereupon lead to false results. It is, therefore, well to note some of these instances by way of general precaution in applying this method.

1. It does not necessarily follow that having observed two forces varying in a constant ratio through several concomitant modifications, the same ratio will be preserved indefinitely through all subsequent changes. Water contracts as it is cooling. Suppose we begin to note this continued contracting of water from 100° F. to 90°; we naturally expect to find it continuing through 90° to 80°. And as we observe, we find our expectations confirmed. And so on through to 40°, we find that water continues to contract. It is, therefore, most natural for us to expect to find water contracting

[1] Darwin, *Voyage of a Naturalist*, Vol. I. pp. 76 f.

at 39°. But just at this point in the series, there is a break in the continuity of variation; at 39° water begins to expand and so continues until it passes into the solid form at the freezing-point. The same also is illustrated in Weber's law, already mentioned, which expresses the quantitative relation between the stimulus and the corresponding sensation. The law is that the force of the stimulus must increase geometrically, in order that the intensity of the sensation should increase arithmetically. This law, however, breaks down towards the upper or lower limits, with a stimulus of slight degree of intensity and with one of extreme intensity. We find also an increase of temperature as we proceed towards the centre of the earth of about one degree to every fifty-three feet of descent. This by no means warrants us in inferring that this ratio continues constant to the very centre itself. In certain phenomena, moreover, there are natural limits, as in sound, for example, where the pitch rises as the number of vibrations increases. At a certain point, varying according to different individuals, increase of vibrations gives no resulting sound whatsoever; and so there is a lower limit, vibrations may decrease to a point beyond which no sound is heard.

An illustration of this fallacy, though not strictly of the method of concomitant variations, is given by Jevons. He takes the following series of prime numbers: 41, 43, 47, 53, 61, 71, 83, 97, 113, 131, etc. It will be seen that they all agree in being values of the general expression $x^2 + x + 41$,

where we put for $x$ the successive values of 0, 1, 2, 3, 4, etc. For instance, let $x = 0$ in $x^2 + x + 41$, we get 41; let $x = 1$ in the same, we get 43; when $x = 2$, we get 47; and so on. It seems as though we could keep this up indefinitely, producing an increasing series, always of prime numbers. It is found, however, that if we take $x = 40$, in the formula $x^2 + x + 41$, we shall have $40 \times 40 + 40 + 41$, which equals 1681, and this number is the square of 41 and therefore not a prime number. At this point the law breaks down.[1]

In the sphere of political economy also we might be led into an easy yet false inference. Suppose a certain farm yield 500 bushels of corn with a given amount of expenditure and labor. We might think that if we doubled the expenditure and labor, we will also be able to double the results, and obtain a yield of 1000 bushels as over against the 500 of the previous year. Here, however, what is known as the law of decreasing returns obtains; to double the product it may be necessary to triple or quadruple the labor and expense. "Thus in the production of any plot of land there is a point of equilibrium, which marks an impassable limit, not of course a limit which could not be passed if it were wished, but one that no one wishes to pass, because there is nothing to be gained by so doing."[2]

To know that such false inferences are at least possible in the application of this method of concomitant variations to the unknown regions beyond

[1] Jevons, *Principles of Science*, p. 230.
[2] Gide, *Political Economy*, p. 325.

our experience, may serve at least to keep us on guard under similar circumstances.

2. There are certain phenomena, moreover, in which an increased intensity of the force in question may give rise to incidental effects which tend to neutralize the chief effect to be attained. For instance, an overdose of arsenic causes violent contractions of the stomach so that its contents are immediately ejected, and thus the system is relieved of the noxious substance.

3. Two elements in a given phenomenon may vary together constantly and yet they may not be related at all as cause and effect, but appear as coincidental effects of one and the same cause. It has been observed that the occurrence of the Aurora Borealis has been accompanied by pronounced magnetic disturbances. It, however, cannot be inferred that the former has been the cause of the latter; they are probably the varied effects of some widely operating magnetic force.

The precaution above mentioned has already been referred to as provided for in the canon of this method which states that the observed concomitant variation may indicate not always a direct causal element between the two varying elements, but that they are at least connected with the phenomenon under investigation through some fact of causation.

## CHAPTER XI

### The Method of Residues

The method of residues consists in the analysis of a given phenomenon based upon previous inductions, through which it has been determined that certain elements in the antecedent have caused certain elements in the consequent; the inference is then drawn, that the remaining elements of the antecedent are necessarily the cause of the remainder of the consequent. It is a method of elimination of the known relations so as to simplify the complex character of the phenomenon and disclose the relations that are unknown in the light of a causal connection which we are constrained to believe must obtain.

*The Canon of the Method of Residues.* — Subduct from any phenomenon such part as is known by previous inductions to be the effect of certain antecedents, and the residue of the phenomenon is the effect of the remaining antecedents.

The symbolical representation is as follows: —

Given $S + C$ . . . . . . . $s + e$

If it is known that there exists the causal relation

$S$ . . . . . . . . . . . . . $s$,

we may then infer that $C$ is the cause of $e$. In this $C$ may be simple or complex; if it is simple, the causal relation established is expressed in its simplest terms and is therefore a determinate result. If, however, the residue $C$ is complex, it must be reduced by experimental analyis to its simplest elements, and their relation to the elements into which $e$ can be analyzed further determined.

The most striking illustration of this method, and one of the most brilliant achievements of science as well, is the discovery of the planet Neptune by Adams and Le Verrier, working on the problem independently and reaching the same result. These astronomers had observed certain perturbations in the planet Uranus. It did not keep in its proper orbit as determined by their mathematical calculations based upon the presence of the known stellar bodies. It behaved as though beyond its orbit was an outer planet, whose presence alone could account for the observed perturbations. Adams and Le Verrier then proceeded to calculate the exact position of such a disturbing body as determined by the nature and magnitude of the perturbations of Uranus. The telescope was then pointed to the exact point in the heavens, as thus indicated, and the planet Neptune was revealed to the eye according to the determination of far-reaching prophecy, which confidently asserted that it must be there.

The method of residues is really a deductive method based upon the law of sufficient reason; so many elements on the one hand producing so many elements on the other; if, then, a part of the former

is to be checked off as cause of a part of the latter, then the remainder on one hand must be the cause of the remainder on the other. This is pure deduction. For we ask, Why are we constrained to account for the remainder on one side by the remainder on the other? The only possible answer is that it *must* be accounted for within the system to which it is referred; and but one part therein is left which can possibly account for it, because all the others are specifically determined in the known effects which they have produced. This method, however, has a proper place among the inductive methods, inasmuch as it is based on previous inductions, and leads to investigations that can be prosecuted only by the various inductive processes of experiment.

When the residue of the antecedent is a simple element, and no other possible causal element can lie concealed from our observation, then the inference is simple and conclusive. A difficulty, however, may present itself, owing to the fact that the residual element is apt to be complex and leave the phenomenon still indeterminate, or there may be a lurking element unnoticed by us which is the real cause in question. The function of this method is, therefore, largely suggestive. It says the effect is not wholly accounted for by the known causal elements; there is a residue unaccounted for, and its cause is to be sought in the residue of the antecedent, and if it is thought that the whole of the antecedent is comprehended, the question is started, May there not be unobserved circumstances of the

antecedent that further experiment will be calculated to reveal? In many cases, therefore, this method must be supplemented by some other experimental method in order to secure more precise determination, generally the method of difference. It often happens in investigations in chemistry, astronomy, and physics, that the actual phenomena vary in greater or less degree from their expected behavior according to established theory. This must lead either to a reconstruction of theory, or to a search for some unobserved force sufficient to account for the discrepancy. Herschel was the first to point out the significance of such discrepancies in scientific research, and he called them *residual phenomena*.

An illustration of such a situation and the solution of the problem thus presented is that of Sir Humphry Davy's experiments upon the decomposition of water by galvanism. "He found that besides the two components of water, oxygen and hydrogen, an acid and alkali were developed at the two opposite poles of the machine. As the theory of the analysis of water did not give reason to expect these products, they were a *residual phenomenon*, the cause of which was still to be found. The insight of Davy conjectured that there might be some hidden cause of this portion of the effect; the glass containing the water might suffer partial decomposition, or some foreign matter might be mingled with the water, and the acid and alkali be disengaged from it, so that the water would have no share in their production. Assuming this, he

proceeded to try whether the total removal of the cause would destroy the effect produced. By the substitution of gold vessels for the glass, without any change in the effect, he at once determined that the glass was not the cause. Employing distilled water, he found a marked diminution of the quantity of acid and alkali evolved; yet there was enough to show that the cause, whatever it was, was still in operation. The impurity of the water, then, was not the sole, but a concurrent cause. He now conceived that the perspiration from the hands touching the instruments might affect the case, as it would contain common salt, and an acid and alkali would result from its decomposition under the agency of electricity. By carefully avoiding such contact, he reduced the quantity of the products still further, until no more than slight traces of them were perceptible. What remained of the effect might be traceable to impurities of the atmosphere decomposed by contact with the electrical apparatus. An experiment determined this; the machine was placed under an exhausted receiver, and when thus secured from atmospheric influence, it no longer evolved the acid and alkali."[1]

By means of the suggestions incident upon this method, Bunsen, in 1860, discovered two new alkaline metals, caesium and rubidium. He was examining alkalies produced by the evaporation of mineral water from Dürkheim. The flame of these salts was examined by the spectroscope. Bunsen discovered several bright lines which he had never

[1] Gore, *The Art of Scientific Discovery*, pp. 432, 433.

noticed before, and which he knew could not be produced by potash or soda, whose corresponding lines were in close proximity. He then subjected the mixture to a searching analysis and succeeded in obtaining two new alkaline substances. When he had separated them, he then tested them by the method of difference, by which he found that they were capable of producing the lines at first noticed; but when withdrawn, the lines instantaneously disappeared.

Thomson and Tait, in their *Elements of Natural Philosophy*, have the following reference and illustration of this method. "When, in an experiment, all known causes being allowed for, there remain unexplained effects (excessively slight it may be), these must be carefully investigated, and every conceivable variation of arrangement of apparatus, etc., tried; until, if possible, we manage so to exaggerate the residual phenomenon as to be able to detect its cause. It is here, perhaps, that in the present state of science we may most reasonably look for extensions of our knowledge; at all events, we are warranted by the recent history of natural philosophy in so doing. Thus, to take only a very few instances, and to say nothing of the discovery of electricity and magnetism by the ancients, the peculiar smell observed in a room in which an electrical machine is kept in action was long ago observed, but called the 'smell of electricity,' and thus left unexplained. The sagacity of Schönbein led to the discovery that this is due to the

formation of ozone, a most extraordinary body, of enormous chemical energies; whose nature is still uncertain, though the attention of chemists has for years been directed to it."[1]

Another illustration of this method is seen in the comparison of the observed and calculated positions of Encke's comet. It was found that the comet returned a little sooner than it should have done, the period regularly decreasing from 1212.79 days, between 1786 and 1789, to 1210.44 between 1855 and 1858. The inference has been that there is a resisting medium, as the ether, filling the space through which the comet passes. What the resisting medium is, and its nature, is of course a matter of conjecture as far as revealed by this method alone. The method merely indicates some resisting medium to account for the observed discrepancy.[2]

Herschel has observed that all great astronomical discoveries have been disclosed in the form of residual differences. The practice was introduced by Halley, when astronomer royal, of comparing systematically the positions of the heavenly bodies as actually observed with what might have been expected theoretically. His reductions of the lunar observations gave a series of residual errors, extending from 1722 to 1739. These were carefully tabulated, and formed the basis for certain modifications of the lunar theory.[3]

[1] Thomson and Tait, *Elements of Natural Philosophy*, Vol. I. pp. 113 f.

[2] Jevons, *Principles of Science*, p. 570. [3] *Ibid.* p. 572.

A discrepancy was observed by Newton between the theoretical and actual velocity of sound; the former being 968 feet per second, and the latter 1142. Newton's experiments and calculation were both inaccurate; nevertheless, a real discrepancy has been proved to exist, the theoretical being 916 and the real velocity 1090 feet per second. In 1816 La Place showed this difference to be due to the heat evolved by the sudden compression of the air during the propagation of the sound wave, the heat having the effect of increasing the elasticity of the air, and therefore appreciably accelerating the sound impulse.

It sometimes happens that in repeating an experiment, we are confronted with evidently different results. Then, we may be sure, the experiment has been carelessly or inaccurately performed; or else there are some disturbing causes not observed by us. On the other hand, however, if there is no likelihood of coincidence on repeated trials, yet, nevertheless, a marked agreement is noticed in the results of various trials, the mind should be at once alert to discover the hidden cause of such agreement, and possibly may be led to new truths of great importance. The following illustration is given by Thomson and Tait: "With a very good achromatic telescope a star appears to have a sensible disc. But, as it is observed that the discs of all stars appear to be of equal angular diameter, we of course suspect some common error. Limiting the aperture of the object-glass increases the appearance in question, which, on full investigation, is

found to have nothing to do with discs at all. It is, in fact, a phenomenon due to diffraction of light."[1]

It was said of Darwin that in his researches the residual phenomena were always the special objects of his attention. His son, Francis Darwin, says of him: "There was one quality of mind which seemed to be of special and extreme advantage in leading him to make discoveries. It was the power of never letting exceptions pass unnoticed. Everybody notices a fact as an exception when it is striking or frequent, but he had a special instinct for arresting an exception. A point apparently slight and unconnected with his present work is passed over by many a man almost unconsciously, with some half-considered explanation, which is in fact no explanation. It was just these things that he seized upon to make a start from. In a certain sense there is nothing special in this procedure, many discoveries being made by means of it. I only mention it, because, as I watched him at his work, the value of this power to an experimenter was so strongly impressed upon me."[2] This is striking testimony as to the practical worth of this method as an instrument of research.

This method has also been applied to the more practical usage of examining the refuse of manufactured and other products in order to discover

[1] Thomson and Tait, *Elements of Natural Philosophy*, Vol. I. p. 114.

[2] F. Darwin, *Life and Letters of Charles Darwin*, Vol. I. p. 125.

some concealed utility. The analysis of coal-tar refuse has led to the discovery of many valuable substances that have proved of use in the arts, and in medicine as well. Glauber, the eminent chemist, and a discoverer of several chemical compounds, said he made it a rule to examine what every other chemist threw away.

## CHAPTER XII

### VERIFICATION AND PREDICTION

*The Inducto-deductive Method.* — We have seen that the inductive methods are efficient in revealing the cause of a given phenomenon under investigation; and yet they do not warrant us in generalizing the special instance so as to formulate a universal law. There is always the possibility that while the special case which we experiment upon may give us indications of an existing causal relation, still a wider experience might disprove, or else modify materially our conclusions. The well-recognized fact of the plurality of causes and the intermixture of effect further embarrasses us in the attempt to rise to a law having universal significance and validity. The results of the inductive methods, therefore, need to be supplemented by some corroborative observations or experiments that will conclusively verify the results as obtained. This supplementary method is one which combines deduction with induction. Mr. Mill calls it the Deductive Method. It is, however, more adequately designated by the name, the Inducto-deductive Method. It consists of three stages: —

1. Obtaining, by the inductive methods already

described, the evidence of some existing causal connection, tentatively expressed in the form of a universal law.

2. Regarding this universal law as the basis for subsequent deductions, by which we gain a knowledge of the nature of unknown phenomena, as necessitated by the conditions of this law.

3. Verifying the results thus obtained by their correspondence with the phenomena as actually observed. Where this correspondence is wanting, then either the law was not correctly expressed, or there must have been some error in our deduction based upon it. When we are assured that the latter is not the case, then a discrepancy between the theoretically deduced result and the actual facts as observed, always discredits our original induction. This method of verification serves as a check upon hasty generalization, on the one hand; and on the other, it serves to extend our knowledge into unknown regions, and is valuable as a means of scientific prediction. In the development of scientific knowledge, it has been a potent factor in enlarging the bounds of knowledge beyond the sphere of immediate observation.

This combined process of reasoning is the one commonly employed by us all. Induction and deduction are not separate processes, but, as before remarked, they are complementary factors in the one actual process of reasoning. We are continually using our inductions as a deductive basis, inferring how things should be before they are really seen; and, when seen, at once instinctively

comparing prior inference with present fact, we are either confirmed in our reasoning process, or compelled to discard our previous inference as false or inadequate as the case may be. Our world, the world of knowledge, is built up of the seen, and the unseen as well, because necessitated by inferences growing out of the seen which we are constrained to make; the unseen which we thus are continually building into the seen and regarding it as though the known, we are, however, from time to time compelled to alter, and here and there tear down what we have too rashly builded up, as the structure is put to the test of verifying fact.

This method of verification was used to decide between inferences drawn by Newton and Huyghens respectively, regarding the nature of light. Newton's observations led him to infer that light consisted of particles of matter shot out from the sun. Huyghens insisted that light consisted in the propagation of some kind of disturbance in the manner of a wave-motion. Newton's theory being taken as established, it would necessitate that light on entering a denser body of water, being refracted more nearly in a direction perpendicular to the surface, should, accordingly, move faster in the denser body than in the rarer one outside. On the other hand, according to Huyghens' theory, the opposite effect should take place, — light being refracted towards the vertical at the horizontal surface of a dense body such as water, its velocity in the dense body should be less than its velocity in

the rare body. The experiments separately made by Fizeau and Foucault, both gave the result that in water light moves slower than in air, and therefore the theory of Huyghens, which was in accord with such a fact, was verified, and the theory of Newton, which was radically out of harmony with such a fact, was discredited.[1]

We cannot theorize concerning nature to any considerable extent without resorting to nature again to correct aberrations of reason, and the false fancies of the imagination. Theory, if correctly formulated, will always lead to a representation of facts as they are; just as facts as they are, if rightly interpreted, will always lead to correct theory.

The following are illustrations of the value of this method in predicting results before unknown.

"Halley had the glory of having first detected a periodic comet in the case of that which has since borne his name. In 1705, Halley explained how the parabolic orbit of a planet may be determined from three observations; and joining example to precept, himself calculated the positions and orbits of twenty-four comets. He found, as the reward of his industry, that the comets of 1607 and 1531 had the same orbit as that of 1682. And here the intervals are nearly the same, namely, about seventy-five years. Are these three comets then identical? In looking back into the history of such appearances, he found comets recorded in 1456, in 1380, and 1305; the intervals are still the same, — sev-

[1] Tait, *Recent Advances in Physical Science*, pp. 65, 66.

enty-five or seventy-six years. It was impossible now to doubt that they were the periods of a revolving body, its orbit a long ellipse, not a parabola. If this were so, the comet must reappear in 1758 or 1759. Halley began his laborious calculations and predicted that the comet would reach its perihelion April 13, 1759, but claimed the license of a month for the inevitable inaccuracies of a calculation in which, in addition to all other sources of error, was made in haste, that it might appear as a prediction. The comet justified his calculations and his caution together; for it arrived at its perihelion on March 13, 1759."[1]

Another illustration of a like nature is the prediction of Faraday, based upon Wheatstone's experimental proof that the conduction of electricity required time; namely, "that if the conducting wires were connected with the coatings of a large Leyden jar, the rapidity of conduction would be necessarily lessened. This prediction was made in 1838 and was not verified until, sixteen years later, a submarine cable was laid beneath the English Channel. A considerable retardation of the electric spark was then detected by Siemens and Latimer Clark. Faraday at once pointed out that the wire surrounded by water resembles a Leyden jar on a large scale: so that each message sent through the cable verified his remark of 1838."[2]

In Pasteur's experiments with silkworms already referred to, he made a prediction in 1866, when,

[1] Whewell, *History of Inductive Science*, 3d ed. Vol. II. p. 182.
[2] Jevons, *Principles of Science*, p. 543.

having inspected fourteen parcels of eggs intended for incubation, and having examined the moths which produced these eggs, he wrote out the prediction of what would occur in 1867, and placed the prophecy as a sealed letter in the hands of the mayor of St. Hippolyte. In 1867, the cultivators communicated to the mayor their results. The letter of Pasteur was then opened and read, and it was found that in twelve out of fourteen cases there was absolute conformity between his prediction and the observed facts. Many of the groups had perished totally; the others had perished almost totally; and such was Pasteur's prediction. In two out of the fourteen cases, instead of the prophesied destruction, half an average crop was obtained.[1]

Another interesting illustration concerns Darwin's speculations regarding the formation of coral reefs and atolls. Before Darwin wrote on the subject, it was generally believed that the coral atolls were formed by the coral polypes growing upon submerged volcanic craters. Darwin insisted that as the polypes cannot live below a depth of 100 feet, and are killed by exposure to sunshine and air, and could not therefore have grown upward from the vast depths to which the coral masses extend, each atoll must have begun as a fringing-reef about an island almost touching the shore, with only a narrow and shallow channel of water between; and then became a barrier reef, that is, one with a wider and deeper channel of water separat-

[1] Tyndall, *Fragments of Science*, pp. 291, 292.

ing from the shore, owing to the slow but progressive subsidence of the island round which the polypes first began to build. Then with the further and complete subsidence of the island beneath the water, there remained a ring of coral with a central lagoon forming the so-called atoll. Darwin says in his Autobiography that the main features of his theory were conceived while on the voyage, and that even previous to seeing a true coral reef.[1] He says: "No other work of mine was begun in so deductive a spirit as this, for the whole theory was thought out on the west coast of South America, before I had seen a true coral reef. I had only to verify and extend my views by a careful examination of living reefs. But it should be observed that I had during the two previous years been incessantly attending to the effects on the shores of South America of the intermittent elevation of the land, together with denudation and deposition of sediment. This necessarily led me to reflect much on the effects of subsidence, and it was easy to replace in imagination the continued deposition of sediment by the upward growth of corals. To do this was to form my theory of the formation of barrier reefs and atolls."

It will thus be seen that Darwin's deduction was based upon previous inductions in other spheres, the result of his own observation; he also tells us in the same connection, that he had, in the preparation of his work on *Coral Reefs*, spent twenty months of hard labor, reading every work on the

[1] *Life and Letters of Charles Darwin*, 1887, Vol. I. p. 58.

islands of the Pacific and consulting many charts. He thus made the widely extended observations of other men tributary to his inferences concerning coral-reef formations. Dr. Williams says of Darwin's insight in this particular: "He saw more clearly than his precursors had done the validity of the dictum of Johannes Müller in this, and indeed all his works, that the most important truths in natural science are to be discovered, neither by the mere analysis of philosophical ideas, nor by simple experience, but by *reflective experience*, which distinguishes the essential from the accidental in the phenomena observed, and thus finds principles from which many experiences can be derived."[1] This is a very satisfactory and striking account of what may be styled the combined inducto-deductive temper of mind, and especially as embodied in so eminent a student of nature as Darwin.

Bacon insists that anticipations of nature are sources of innumerable errors, and that the truly scientific method consists in an interpretation of nature as it is revealed to the perception through direct observation and experiment. It is, however, largely through these "anticipations" that progress in science is attained. There may be anticipations which are considered final, and all attempts at verification regarded as unnecessary and even as impertinent. Results deductively attained are then asserted with dogmatic insistence, as though pos-

[1] Darwin, *Coral Reefs*. Prefatory note by Dr. J. W. Williams, p. ix.

sessing the convincing power of facts themselves; and all appeal to controverting or exceptional cases are set aside, without even so much as a respectful hearing. Such anticipations of nature rightfully fall under the scornful reprehension of a Bacon. But there are other anticipations which serve as a spur to a more penetrating observation, and more painstaking experiment, in order to square theory to facts. Such anticipations are the glory of science!

Suppose such anticipations are disproved by subsequent experiment or observation; they have served a high purpose in suggesting investigation along lines which otherwise would have remained unthought of. Anticipations alone are barren; anticipations leading to verification are productive of valuable results. To this the history of scientific thought bears abundant testimony. Professor Clifford has made the power of prediction one of the essential characteristics of scientific thought. He says, in his essay on the *Aims and Instruments of Scientific Thought,* that "the difference between scientific and merely technical thought is just this: Both of them make use of experience to direct human action; but while technical thought or skill enables a man to deal with the same circumstances that he has met with before, scientific thought enables him to deal with different circumstances that he has never met with before."[1] He cites two illustrations, which are admirable examples of scientific prediction. The first relates to the suggestion of Fleeming Jenkin, regarding structural bracing. It

[1] Clifford, *Lectures and Essays,* Vol. I. p. 128.

had been known before that in an arch every part is compressed or pushed by other parts; and every part of a chain is in a state of tension, that is, pulled by the other parts. In many cases these forms are united in the common girder, which consists of two main pieces, of which the upper acts as an arch, and is compressed, while the lower one acts as a chain and is pulled. "Now," says Professor Clifford, "suppose that any good, practical engineer makes a bridge or a roof upon some approved pattern which has been made before. He designs the size and shape of it to suit the opening which has to be spanned; selects his material according to the locality; assigns the strength which must be given to the several parts of the structure, according to the load which it will have to bear. There is a great deal of thought in the making of this design, whose success is predicted by the application of previous experience; it requires technical skill of a very high order, but it is not scientific thought. On the other hand, Mr. Fleeming Jenkin designs a roof consisting of two arches braced together, instead of an arch and a chain braced together; and, although this form is quite different from any known structure, yet before it is built he assigns with accuracy the amount of material that must be put into every part of the structure in order to bear the required load, and this prediction may be trusted with perfect security. What is the natural comment on this? Why, that Mr. Fleeming Jenkin is a *scientific* engineer."[1]

[1] Clifford, *Lectures and Essays*, Vol. I. pp. 127, 128.

The second illustration which Professor Clifford gives is as follows: "You know that if you make a dot on a piece of paper, and then hold a piece of Iceland spar over it, you will see not one dot, but two. A mineralogist, by measuring the angles of a crystal, can tell you whether or no it possesses this property without looking through it. He requires no scientific thought to do that. But Sir William Rowan Hamilton, the late astronomer royal of Ireland, knowing these facts, and also the explanation of them, which Fresnel had given, thought about the subject, and he predicted that by looking through certain crystals in a particular direction we should see not two dots, but a continuous circle. Mr. Lloyd made the experiment and saw the circle, a result which had never been even suspected. This has always been considered one of the most signal instances of scientific thought in the domain of physics. It is most distinctly an application of experience gained under certain circumstances to entirely different circumstances."[1]

There is also an indirect method of prediction, varying somewhat from the one already described and yet similar to it. It is called prediction by inversion of cause and effect. There are many cases in which cause and effect are related in a reciprocal manner, so that not only will the cause produce the effect, but the effect, operating as a cause, will bring about the original cause as an effect, it may be in a modified form but clearly recognizable as such. Professor Tyndall said of Faraday

[1] Clifford, *Lectures and Essays*, Vol. I. pp. 128, 129.

that "the strong tendency of his mind to look upon the reciprocal actions of natural forces gave birth to his greatest discoveries."[1] For instance, Oersted had proved that an electric current will produce magnetism, and Faraday, taking this as a suggestion, inferred that magnetism might produce an electric current; in the year 1831 he devised a suitable experiment of introducing a bar-magnet into a coil of insulated copper wire, and then withdrawing the magnet whilst the two ends of the wire were connected with a distant galvanometer, which indicated the presence of the electric current. Thus, his inference received substantial verification.[2]

It has, moreover, been found that when a given cause produces a certain effect, then if the effect be produced in some other manner, the process will tend to produce the original cause, but inverted as regards its direction or nature. For instance, it is known that heat will expand gases; now, if a gas be relieved of the pressure of the vessel enclosing it, it will expand by virtue of its own elastic power, producing, however, cold in the surrounding atmosphere. So also heat will cause a bar of iron to expand. Dr. Joule proved that if iron were expanded by mechanical force, it would be accompanied by cold. Inasmuch as india-rubber is related to heat in an opposite manner to that of iron, being contracted by heat instead of expanded, we would, according to the law above expressed, naturally expect that a mechanical expansion of india-rubber would give

[1] Tyndall, *Fragments of Science*, p. 338.

[2] Gore, *The Art of Scientific Discovery*, p. 594.

heat, and a contraction produce cold. An experiment may be tried by suddenly stretching a rubber band while the middle part is in the mouth; when stretched, it grows warm; when relaxed, it seems cold.[1]

Again, as heat will melt many substances, if we can reduce the same substance from the solid to the liquid state, we would expect, as a result, the negative of heat, namely, cold. This occurs in all freezing mixtures, as the affinity of salt for water causes it to melt ice, thus producing cold in the surrounding atmosphere, sufficient to freeze cream or other similar substance, inasmuch as, passing from solid to liquid, water absorbs heat from all substances near it; this absorption producing artificial cold surrounding it. The reciprocal action of heat and cold is illustrated in an interesting experiment described by Tait.[2] He took a bar of ice, supported horizontally at either end, and over the middle of the bar he put a fine wire, and put equal weights to the two ends of the wire. The wire gradually, by the action of the weights, cut through the bar of ice, and yet it was observed that the path of the wire was instantly replaced by the freezing again of the melted portion produced by the pressure, and when the wire had wholly traversed the entire thickness of the bar, the bar itself was intact, and even stronger along the line of the cutting than before. The explanation of this experiment is that inasmuch as heat melts ice, then when ice is melted

[1] Jevons, *Principles of Science*, p. 545.

[2] Tait, *Recent Advances in Physical Science*, pp. 99, 100.

by pressure, as in this case of the weighted wire, cold, the negative of heat, is induced; thus, as the wire was forced by the weights into the ice, the pressure upon the ice melted it, making it colder, so that the water produced, passing around the chilled wire, and being thus relieved of pressure, froze again.

Faraday predicted certain magnetic phenomena by this method, which are specially interesting as illustrations of this kind of prediction. It seems that Arago had observed in 1824 that the number of oscillations which a magnetized needle makes in a given time, under the influence of the earth's magnetism, is very much lessened by the proximity of certain metallic masses, and especially of copper. Employing the latter substance in an experiment upon a magnetized needle, he succeeded in reducing the number of its vibrations in a given time from three hundred to four. Taking the experiment as a basis for his inference, Faraday predicted that since the presence of a metal at rest stops the oscillations of a magnetic needle, the neighborhood of a magnet at rest ought to stop the motion of a rotating mass of metal. He therefore proceeded to put his inference to the test of actual experiment, by suspending a cube of copper to a twisted thread which was placed between the poles of a powerful electromagnet. When the thread was left to itself, it began to spin round with great velocity, but stopped the moment a powerful current passed through the electromagnet.[1] Again, as heat applied to the junction of

[1] Ganot, *Physics*, pp. 797, 798.

two metallic bars, as antimony and bismuth, produced an electric current, it was inferred that if an electric current was made to pass through such a junction, it would produce cold, and such proved to be the case.[1]

In the general process of verification, it often happens that seeming exceptions occur which are direct contradictions of the law we are attempting to prove. And it is in dealing with such cases that one's power of discrimination is most fully taxed. It is necessary to make a most careful distinction between seeming and real exceptions. Professor Jevons has given a very exhaustive classification of the different kinds of exceptional phenomena, which it is well to have in mind, in order to know in any investigation the various possible complications that may rise.[2] The exceptional phenomena, as given by Jevons, are: —

1. Imaginary, or false exceptions; that is, facts, objects, or events which are not really what they are supposed to be.

2. Apparent but congruent exceptions, which, though apparently in conflict with a law of nature, are really in agreement with it.

3. Singular exceptions, which really agree with a law of nature, but exhibit remarkable and unique results of it.

4. Divergent exceptions, which really proceed from the ordinary action of known processes of

[1] Jevons, *Principles of Science*, p. 547.

[2] See Jevons, Chapter XXIX., in his *Principles of Science*, on "Exceptional Phenomena."

nature, but which are excessive in amount or monstrous in character.

5. Accidental exceptions, arising from the interference of some entirely distinct but known law of nature.

6. Novel and unexplained exceptions, which lead to the discovery of a new series of laws and phenomena, modifying or disguising the effects of previously known laws without being inconsistent with them.

7. Limiting exceptions, showing the falsity of a supposed law to some cases to which it had been extended, but not affecting its truth in other cases.

8. Contradictory, or real, exceptions, which lead us to the conclusion that a supposed hypothesis or theory is in opposition to the phenomena of nature, and must therefore be abandoned.

It will be seen that among so many possibilities of interpretation an exception does not necessarily prove the rule, as the old adage would have it; nor does the exception, on the other hand, necessarily disprove the rule or law. It must be in each case strictly and adequately interpreted, which requires a penetrating sagacity and a thorough knowledge of the phenomena under investigation.

In the process of verification, the question naturally suggests itself: How many verifying instances are sufficient to determine the universal validity of a given law? This question will be recognized as an old difficulty, now presented in another form; but in reality it is the perplexing problem of determining the logical ground of in-

duction. What is our warrant for proceeding from known and verified instances to unknown phenomena, of the same kind it is true, but as yet beyond the pale of our experience? The warrant for our generalization does not lie wholly in the number of verifying instances. In addition to the effect which mere number produces in confirming our belief, there is the confidence which we feel in the constancy of the order of nature, and which we are constrained to assume as a fundamental postulate.[1] Therefore, we say that the verifying facts must be of such a number, and of such a nature as well, that they give evidence of a uniformity which transcends all supposition of mere coincidence, and compels us to attribute it to the uniformity of nature itself, in which we find a warrant for our generalization. As Professor Clifford has remarked: "The aim of scientific thought is to apply past experiences to new circumstances. The instrument is an observed uniformity in the course of events. By the use of this instrument it gives us information transcending our experience, it enables us to infer things that we have not seen from things that we have seen; and the evidence for the truth of that information depends on our supposing that the uniformity holds good beyond our experience." [1]

In extending knowledge and predicting results beyond the sphere of experience, modern scientific investigation is largely indebted to the principles and methods of mathematics. Mathematical laws,

[1] See Sigwart, *Logic*, Vol. II. p. 348.

[2] Clifford, *Lectures and Essays*, Vol. I. pp. 131, 132.

applied to the data given in sense-perception, give indications of the necessary relations that must exist in the observed phenomena, and all that they involve. Thus, that which is given directly in consciousness is supplemented by that which is given indirectly as mathematically necessitated. The mathematico-experimental method in physics has led to very rich and important results which have proved practically its efficiency as a scientific method.

# CHAPTER XIII

## Hypothesis

The inductive process cannot proceed to any great extent or attain satisfactory results without the aid of some hypothesis. An hypothesis is a supposition regarding the cause of a phenomenon, which we make either as preliminary to an experiment which will prove or disprove the supposition, or in lieu of an experiment or systematic observation when such are impossible, owing to the peculiar conditions of the phenomenon itself. We see, therefore, that the framing of hypotheses has a double function. First, considered as preliminary to experiment. We found that in cases where two, three, or more elements enter into a complex antecedent, it is impossible often, and always impracticable to test the various possible combinations separately in order to note their different results. The combinations in complex phenomena are indefinitely great, and the isolation of certain elements in order to estimate the exact result of the combined force of the other combinations is extremely difficult and often impossible. Therefore the mind discards some combinations as irrelevant, others as impossible, and selects one perhaps as the most likely cause of the

given effect. This selective function of the mind, therefore, indicates the line of experiment in a determinate manner and does not leave the phenomena to indeterminate and haphazard investigation.

Consider, for instance, so eminent an experimenter as Charles Darwin, so fertile in all kinds of experimental resources; yet it is said of him that every experiment was the result of a tentative theory, thought out in advance of all actual test by a sagacious insight into the necessary conditions of the interrelated phenomena before him. His son, Francis Darwin, says of him in his *Reminiscences*: "He often said that no one could be a good observer unless he was an active theorizer. It was as though he were charged with theorizing power ready to flow into any channel on the slightest disturbance, so that no fact, however small, could avoid releasing a stream of theory, and thus the fact became magnified into importance. In this way, it naturally happened that many untenable theories occurred to him; but fortunately his richness of imagination was equalled by his power of judging and condemning the thoughts that occurred to him. He was just to his theories, and did not condemn them unheard; and so it happened that he was willing to test what would seem to most people not at all worth testing. These rather wild trials he called 'fool's experiments,' and enjoyed extremely. As an example, I may mention that, finding the cotyledons of Biophytum to be highly sensitive to vibrations of the table, he fancied that they might perceive the vibrations of sound, and therefore made me play

my bassoon close to a plant. The love of experiment was very strong in him, and I can remember the way he would say, 'I shan't be easy till I have tried it,' as if an outside force were driving him."[1]

Hypothesis and experiment were in the hand of Darwin like a two-edged sword, which he employed with rare skill and effect. An hypothesis is to be regarded not only as the precursor of experiment, but it also functions as a method of explanation when actual verification is impossible. We see this constantly in our every-day life. We are compelled again and again to account for situations which occur that are impossible for us to reproduce in the form of an experiment, that we are able to observe but once. Some explanation is required to satisfy mental demands which are imperative in this regard. The explanation which seems most in keeping with the sum of facts in our possession, is the hypothesis which we frame; so also in explaining the conduct of others by conjecture as to the most reasonable motives that will satisfactorily account for the same; such hypotheses we are constantly compelled to assume. We are not always able to perceive the relations existing between facts as they come into the sphere of our experience, and yet we are constrained to think of them as related; but in order to systematize them, we must supply mentally the *lacunæ* which appear in the phenomena as perceived. This supposition that is necessary to construct facts into system is an hypothesis.

[1] *Life and Letters of Charles Darwin*, Vol. I. p. 126.

An illustration of an hypothesis suggesting systematic observation and experiment is found in the history of the discovery of vaccination by Jenner. It seems that while a mere youth, pursuing his studies at Sodbury, he chanced to hear a casual remark made by a country girl who came to his master's shop for advice. The small-pox was mentioned, when the girl said, "I cannot take that disease, for I have had cow-pox."[1] This observation, expressing the common superstition of the simple country folk, appealed to Jenner's mind as an inchoate hypothesis. Seizing upon it as a suggestion of possible value, he proceeded to make diligent inquiries and careful observations, which finally led him to the discovery of vaccination.

An illustration of hypothesis as explanation of phenomena beyond the range of experiment is found in the hypothesis as to the source of the sun's energy. An enumeration of the different hypotheses advanced upon this subject is given by Tait in his *Recent Advances in Physical Science*.[2] "The old notion that the sun is a huge fire, or something of that kind, is one which will only occur to one thinking of the matter for the first time; but with our modern chemical knowledge, we are enabled to say that, massive as the sun is, if its materials had consisted of the very best materials for giving out heat, that enormous mass of some 400,000 miles in radius could have supplied us with only about 5000 years of the present radiation. A mass of

[1] Gore, *The Art of Scientific Discovery*, p. 495.
[2] pp. 151 ff.

coal of that size would have produced very much less than that amount of heat. Nor would the most energetic chemicals known to us, combined in proportion for giving the greatest amount of heat by actual chemical combination, supply the sun's present waste for even 5000 years. Therefore as we all know that geological facts, if there were no others, point to at least as high a radiation from the sun as the present, for at all events a few hundreds of thousands of years back, — and perhaps also indicate even a higher rate of radiation from the sun in old time than at present, — it is quite obvious that the heat of the sun cannot possibly be supplied by any chemical process of which we have the slightest conception.

"Now, if we can find, on the other hand, any physical explanation of this consistent with any present knowledge, we are bound to take it and use it as far as we can, rather than say: This question is totally unanswerable unless there be chemical agencies at work in the sun of a far more powerful order than anything we meet with on the earth's surface. If we can find a thoroughly intelligible source of heat, which, though depending upon a different physical cause from the usual one, combustion, is amply sufficient to have supplied the sun with such an amount of heat as to enable it to radiate for perhaps the last hundred millions of years at the same rate as it is now radiating, then I say we are bound to try that hypothesis first, and argue upon it until we find it inconsistent with something known. And if we

do not find it inconsistent with anything that is known, while we find it completely capable of explaining our difficulty, then it is not only philosophic to say that it is most probably the origin of the sun's energy, but we feel ourselves constrained to admit it. Newton long ago told us this obligation in his *Rules of Philosophizing.* Now it is known that if we were to take a mass of the most perfect combustibles which we know, and let it fall upon the sun merely from the earth's distance, then the work done upon it by the sun's attraction during its fall would give it so large an amount of kinetic energy when it reached the sun's surface as to produce an impact which would represent six thousand times the amount of energy which could be produced by its mere burning.

"It appears, then, that our natural and only trustworthy mode of explaining the sun's heat at present, in time past, and for time to come must be something closely analogous to, but not identical with, what was called the nebular hypothesis of Laplace, — the hypothesis of the falling together (from rudely scattered distribution in space) of the matter which now forms the various suns and planets. We find by calculation in which there is no possibility of large error, that this hypothesis is thoroughly competent to explain one hundred millions of years' solar radiation at the present rate, perhaps more; and it is capable of showing us how it is that the sun, for thousands of years together, can part with energy at the enormous

rate at which it does still part with it, and yet not apparently cool by perhaps any measurable quantity.

"In confirmation of this, not only is the hypothesis itself capable of explaining the amounts of energy which are in question, but also recent investigations, aided by the spectroscope, have shown us that there are gigantic nebular systems at great distances from our solar system, in the process of physical degradation in that very way, by the falling together of scattered masses, and with numerous consequent developments of heat by impacts. What are called temporary stars form another splendid and still more striking instance of it, as where a star suddenly appears, of the first magnitude, or even brighter than the first, outshining all the planets for a month or two at a time, and then, after a little time, becomes invisible in the most powerful telescope. Things of that kind are constantly occurring on a larger or smaller scale and they can all be easily explained on this supposition of the impact of gravitating masses."

Such a hypothesis, it will be seen, embraces all the facts observed in one self-consistent system. The other hypotheses are inadequate to account satisfactorily for the phenomena. The validity of this hypothesis lies in its being both adequate and congruent as well; experiment or corroborative observation being out of the question, we are, as Tait says, "constrained to admit it."

Mr. Darwin gives an enumeration and criticism

of the different hypotheses which have been suggested to explain the extinction of the gigantic animals known to have existed upon the earth. His account will give an indication of the natural propensity of the mind to frame hypotheses concerning phenomena which lie outside the sphere both of observation and experiment. Mr. Darwin says: "It is impossible to reflect on the changed state of the American Continent without the deepest astonishment. Formerly it must have swarmed with great monsters; now we find mere pigmies compared with the antecedent allied races. The greater number, if not all, of these extinct quadrupeds, lived at a late period, and were the contemporaries of most of the existing sea-shells. What then has exterminated so many species and whole genera? The mind at first is irresistibly hurried into the belief of some great catastrophe; but thus to destroy animals, both large and small, in Southern Patagonia, in Brazil, on the Cordillera of Peru, in North America up to Behring's Straits, we must shake the entire framework of the globe.

"An examination, moreover, of the geology of La Plata and Patagonia leads to the belief that all the features of the land result from slow and gradual changes. It appears from the character of the fossils in Europe, Asia, Australia, and in North and South America, that those conditions which favor the life of the *larger* quadrupeds were lately coextensive with the world. What those conditions were, no one has yet even conjectured. It could hardly have been a change of

temperature, which at about the same time destroyed the inhabitants of tropical, temperate, and arctic latitudes on both sides of the globe. In North America we positively know from Mr. Lyell that the large quadrupeds lived subsequently to that period when boulders were brought into latitudes at which icebergs now never arrive; from conclusive but indirect reasons we may feel sure that in the southern hemisphere the Macrauchenia also lived long subsequently to the ice-transporting boulder-period. Did man, after his first inroad into South America, destroy, as has been suggested, the unwieldy Megatherium and the other Edentata? We must look at least to some other cause for the destruction of the little tucutuco at Bahia Blanca, and of the many fossil mice and other small quadrupeds in Brazil. No one will imagine that a drought, even far severer than those which cause such losses in the provinces of La Plata, could destroy every individual of every species from Southern Patagonia to Behring's Straits. What shall we say of the extinction of the horse? Did those plains fail of pasture which have since been overrun by thousands and hundreds of thousands of the descendants of the stock introduced by the Spaniards? Have the subsequently introduced species consumed the food of the great antecedent races? Can we believe that the Capybara has taken the food of the Toxodon, the Guanaco of the Macrauchenia, the existing small Edentata of their numerous gigantic prototypes? Certainly no fact in the long history of the world

is so startling as the wide and repeated exterminations of its inhabitants."[1] Mr. Darwin's own hypothesis concerning this phenomenon is rather indefinite, but nevertheless as definite as the extreme complexity of the facts will allow. He says that there are certain causes operating in nature, their exact character remaining unknown, such that the too rapid increase of every species, even the most favored, is steadily checked, producing in some cases rarity and in others extinction, if these causes operate with unusual efficacy. His hypothesis marks a tendency whose nature, nevertheless, remains concealed.

In all these widely differing hypotheses we see a certain mental constraint to offer some explanation, even though it be but a disguised confession of ignorance, as in Mr. Darwin's hypothesis.

An illustration of an hypothesis to explain observed phenomena that cannot be further tested is that given in the following instance cited by Professor Tyndall: "At Erith, in 1864, there occurred a tremendous explosion of a powder magazine. The village of Erith was some miles distant from the magazine, but in nearly all cases the windows were shattered; and it was noticeable that the windows turned away from the origin of the explosion suffered almost as much as those which faced it. Lead sashes were employed in Erith church; and these, being in some degree flexible, enabled the windows to yield to pressure without much fracture of glass. Every window in the church, front and back, was

[1] Darwin, *Voyage of a Naturalist*, Vol. I. p. 223.

bent *inwards*. In fact, as the sound-wave reached the church, it separated right and left, and, for a moment, the edifice was clasped by a girdle of intensely compressed air, which forced all its windows inwards. After compression, the air in the church no doubt dilated, and tended to restore the windows to their first condition. The bending in of the windows, however, produced but a small condensation of the whole mass of air within the church; the force of the recoil was, therefore, feeble in comparison with the force of impact, and insufficient to undo what the latter had accomplished."[1] Here also is a set of conditions that must be satisfied by a correct hypothesis. The phenomenon was not capable of repetition by any experiment. Professor Tyndall, therefore, pictures to his mind what must have happened beyond that which was observed, in order to account for the result which actually happened. He fills up the unseen from what he knows of the nature of sound-waves, and thus constructs one self-consistent system which includes both the seen and the unseen, the known and the unknown, the observed and the inferred.

It will be noticed in this and other illustrations of hypothesis, how large a part is played by the imagination. It is the imagination which fills out the vacant spaces in the picture of perception. With some, the function of imagination is associated with fancy rather than fact. It must, in this connection, however, be clearly emphasized

[1] Tyndall, *On Sound*, p. 23.

that the imagination which constructs hypotheses must be throughout in touch with fact. It must represent to the mind, not what fancy suggests, but what the known facts necessitate. The unseen is constructed out of the determining conditions of the seen. It is this deductive function of the imagination that gives to it a strictly logical significance. For instance, Professor Tyndall's reasoning concerning the Erith church was somewhat as follows: The windows are all bent inward, therefore the pressure must have operated on all sides from without, inward; such pressure could only occur upon the supposition that the sound-waves, separating right and left, wholly encompassed the church, etc. In each case, that which he pictured to his mind as happening, was regarded by him as actually necessitated by the facts as observed.

Professor Tyndall has most admirably discussed the "Scientific Use of the Imagination;" and his lecture under that title every student, both of logic or of science, should read. I quote one passage from it, which has special bearing upon what has just been said: "We are gifted with the power of Imagination, — combining what the Germans call *Anschauungsgabe* and *Einbildungskraft*, — and by this power we can lighten the darkness which surrounds the world of the senses. There are tories in science who regard imagination as a faculty to be feared and avoided rather than employed. They had observed its action in weak vessels and were unduly impressed by its disasters. But they might with equal justice point to exploded boilers as an

argument against the use of steam. Bounded and conditioned by co-operant Reason, imagination becomes the mightiest instrument of the scientific discoverer. Newton's passage from a falling apple to a falling moon was, at the outset, a leap of the imagination. When William Thomson tries to place the ultimate particles of matter between his compass points, and to apply to them a scale of millimetres, he is powerfully aided by this faculty. And in much that has been recently said about protoplasm and life, we have the outgoings of the imagination guided and controlled by the known analogies of science. In fact, without this power our knowledge of nature would be a mere tabulation of coexistences and sequences. We should still believe in the succession of day and night, of summer and winter; but the soul of Force would be dislodged from our universe; causal relations would disappear, and with them that science which is now binding the parts of nature to an organic whole."[1]

In all the illustrations which have been given, and, in fact, in all examples of the framing of hypotheses, it will be seen that the mental functions specially in operation are those of analysis and synthesis, — a separation of the elements as far as possible into their simplest forms of expression, and the building them together into some one system whose unity lies in the assumed hypothesis. Mr. Venn has especially emphasized this aspect of

[1] Tyndall, *Use and Limit of the Imagination in Science*, p. 16.

hypothesis, and his chapter on this subject will well repay a careful reading.[1]

Every supposition, however, is not necessarily an hypothesis in the logical or scientific significance of that term. It will be necessary, therefore, to mention some of the requirements which a logical hypothesis should satisfy.

1. An hypothesis should be plausible; that is, it should be no fanciful, or merely conjectural, explanation of the phenomena in question. The suppositions of the interference of spirits, or in a mythological age of the gods, to account for perplexing situations or obscure happenings, have no rank as hypotheses; so, also, Fate is often referred to as a convenient confession of ignorance in lieu of a satisfactory explanation. Spinoza has remarked upon this as follows: "They who have desired to find scope for the display of their ingenuity in assigning causes, have had recourse to a new style of argument to help them in their conclusions, namely, by reduction, not to the impossible or absurd, but to ignorance or the unknown, a procedure which shows very plainly that there was no other course open to them."

The difference between a scientific hypothesis and a popular explanation concerning the same phenomena may be found in Darwin's account of "a strange belief which is general amongst the inhabitants of the Maldiva atolls, namely, that corals have roots, and therefore that if merely broken down to the surface, they grow up again;

[1] Venn, *Empirical Logic*, Chapter XVI.

but if rooted out, they are permanently destroyed. By this means the inhabitants keep their harbors clear; and thus the French governor of St. Mary's in Madagascar 'cleared out and made a beautiful little port at that place.'"[1] Their explanation, however, is purely fanciful, having no basis in fact. In contrast, Darwin's hypothesis to explain the facts in the case is of a logically scientific nature, and is as follows: Inasmuch as loose sediment is injurious to the living polypifers, and as it is probable that sand would accumulate in the hollows formed by tearing out the corals, but not on the broken and projecting stumps, therefore in the former case the fresh growth of coral might be thus prevented by the deposited sediment.

2. The second requirement is that the hypothesis must be capable of proof or disproof. This does not demand a test by experiment necessarily; for that, as we have seen, may be impossible. It does, however, require that some facts should be forthcoming that will either confirm the hypothesis or disprove it. There are cases, however, as Lotze suggests, whose very nature precludes the possibility of proving or disproving the hypothesis framed to account for them. For instance, the very common and simple hypothesis of regarding the stars, which are apparently but small points of light, as bodies of vast size, only very remote from us, is in itself incapable of being either refuted or confirmed by subsequent discovery. Lotze says: "We must abide content if our hypotheses are

[1] Darwin, *Coral Reefs*, p. 89.

thinkable and useful, if they are capable of explaining all interconnected appearances, even such as were still unknown when we constructed them, if, that is to say, they are indirectly confirmed by the agreement of all that can be deduced from them in thought with the actual progress of experience. But if we would be so fortunate as to find an hypothesis which will not lack this subsequent confirmation, we must not simply assume anything that can be barely conceived as real; we must only assume that which, besides being thinkable, conforms, so to speak, to the universal customs of reality, or to the special local customs which prevail in that department of phenomena to which the object we are investigating belongs."[1]

It is to be specially observed that while the requirement of proof of an hypothesis may be waived in the sphere of phenomena where proof is manifestly impossible, still, where proof is available, an hypothesis must never be so framed as to render the required test either impossible or impracticable.

3. The hypothesis must be adequate. It must cover all the facts in the case. An outstanding fact which it cannot explain is sufficient to controvert such an hypothesis. A knowledge of the distinction between postulate and hypothesis, and of the relation which nominally exists between the two, will help us to appreciate more clearly the force of this requirement of adequacy. As defined by Lotze, a postulate "expresses the conditions which must be set up, or the ground of explanation which

[1] Lotze, *Logic*, p. 353.

must be given by some reality, force, or event, before we can think the phenomenon in the form in which it is presented to us; it thus requires or postulates the presence of something that can account for the given effect. An hypothesis is a conjecture which seeks to fill up the postulate thus abstractly stated by specifying the concrete causes, forces, or processes out of which the phenomenon really arose in this particular case, while in other cases maybe the same postulate is to be satisfied by utterly different though equivalent combinations of forces or active elements."[1] According to this distinction as applied to the problem of the source of the sun's energy, the postulate would be the sum of conditions which require explanation; namely, the tremendous radiation of heat extending through thousands and thousands of years. The postulate therefore requires a force adequate to supply for so long a period so great an amount of energy. We found that ordinary combustion of the most highly combustible materials would not, as an hypothesis, satisfy the conditions which obtain in the postulate; nor would the liberation of chemical energy stand as an hypothesis adequate to satisfy the postulate; the hypothesis of impact of masses upon the sun's surface from immense distances presents a force sufficient to meet the requirements of the postulate. Moreover, we see in this illustration how the hypothesis is a particular and concrete expression of the conditions expressed in general and in abstract terms in the postulate. The essential characteristic

[1] Lotze, *Logic*, pp. 349, 350.

therefore of the hypothesis is that it shall perfectly satisfy all the conditions expressed in the postulate.

The hypothesis that nature abhorred a vacuum, in order to account for the rise of water in a tube or pump, was seen to break down utterly when it was found that the water did not rise beyond some thirty-three feet. The demand of the postulate in the case was a force of precisely such magnitude that it would balance a column of water thirty-three feet in height. This force, precisely satisfying the conditions of the postulate, is found in the hypothesis that the atmospheric pressure is such a magnitude as to exert a pressure equivalent to a column of water some thirty-three feet in height. The strength of the hypothesis lies in its exact and appropriate fitting into the facts of the problem.

Another illustration of the fitting of hypothesis to postulate, and one where the conditions of the postulate are extremely complex, I have chosen from Mr. Wallace's work, *On Natural Selection:* "There is a Madagascar orchis — the *Angræcum sesquipedale* — with an immensely long and deep nectary. How did such an extraordinary organ come to be developed? Mr. Darwin's explanation is this. The pollen of this flower can only be removed by the base of the proboscis of some very large moths, when trying to get at the nectar at the bottom of the vessel. The moths with the longest probosces would do this most effectually; they would be rewarded for their long tongues by getting the most nectar; whilst, on the other hand, the flowers with the deepest nectaries would be the best

fertilized by the largest moths preferring them. Consequently the deepest-nectaried orchids and the longest-tongued moths would each confer on the other an advantage in the battle of life. This would tend to their respective perpetuation, and to the constant lengthening of nectaries and probosces. In the *Angræcum sesquipedale* it is necessary that the proboscis should be forced into a particular part of the flower, and this would only be done by a large moth burying its proboscis to the very base, and straining to drain the nectar from the bottom of the long tube, in which it occupies a depth of one or two inches only. Now let us start from the time when the nectary was only half its present length, or about six inches, and was chiefly fertilized by a species of moth which appeared at the time of the plant's flowering, and whose proboscis was of the same length. Among the millions of flowers of the Angræcum produced every year, some would always be shorter than the average, some longer. The former, owing to the structure of the flower, would not get fertilized, because the moths could get all the nectar without forcing their trunks down to the very base. By this process alone the average length of the nectary would annually increase, because the short-nectaried flowers being sterile, and the long ones having abundant offspring, exactly the same effect would be produced as if a gardener destroyed the short ones, and sowed the seed of the long ones only; and this we know by experience would produce a regular increase of length, since it is this

very process which has increased the size and changed the form of our cultivated fruits and flowers. But this would lead in time to such an increased length of the nectary that many of the moths could only just reach to the surface of the nectar, and only the few with exceptionally long trunks be able to suck up a considerable portion. This would cause many moths to neglect these flowers, because they could not get a satisfying supply of nectar, and if these were the only moths in the country the flowers would undoubtedly suffer, and the further growth of the nectary be checked by exactly the same process which had led to its increase.

"But there are an immense variety of moths, of various lengths of proboscis, and as the nectary became longer, other and larger species would become the fertilizers, and would carry on the process till the largest moths became the sole agents. Now, if not before, the moth would also be affected; for those with the longest probosces would get the most food, would be the strongest and most vigorous, would visit and fertilize the greatest number of flowers, and would leave the largest number of descendants. The flowers most completely fertilized by these moths being those which had the longest nectaries, there would in each generation be, on the average, an increase in the length of the nectaries, and also an average increase in the length of the probosces of the moths; and this would be a *necessary result* from the fact that nature ever fluctuates about a mean, or that in every generation

there would be flowers with longer and shorter nectaries, and moths with longer and shorter probosces than the average. I may here mention that some of the large Sphinx moths of the tropics have probosces nearly as long as the nectary of *Angræcum sesquipedale*. I have carefully measured the proboscis of a specimen of *Macrosila cluentius* from South America, in the collection of the British Museum, and find it to be nine inches and a quarter long. One from tropical Africa (*Macrosila morganii*) is seven inches and a half. A species having a proboscis two or three inches longer could reach the nectar in the longest flowers of *Angræcum sesquipedale*, whose nectaries vary in length from ten to fourteen inches. That such a moth exists in Madagascar may be safely predicted;[1] and naturalists who visit that island should search for it with as much confidence as astronomers searched for the planet Neptune, — and I venture to predict they will be equally successful."[2]

I have given this quotation at length in order to indicate not only the fitting of hypothesis to the facts observed, but also the large and important part performed by the imagination in reproducing along parallel lines the natural history of the orchid and moth. The hypothesis reaches back over an indefinitely long past, by virtue of the necessities

[1] It is interesting to note that since Mr. Wallace wrote the above, Kirby, in his *European Moths and Butterflies*, makes mention of one of the Sphingidæ with a proboscis *twelve* inches long!

[2] Wallace, *On Natural Selection*, pp. 271–275.

observed in the present, and in accordance with well-established analogies and approved inductions. The function of the imagination especially prominent is that of its deductive insight, which is able to picture to the mind the inevitable results of this and that condition as furnished by the postulate, and then to fit such necessitated results into one self-consistent system, with nothing left unexplained, incongruous, or contradictory.

Another illustration of an hypothesis covering a large number of complex facts is that of the fertilization of certain flowers by means of the wind. As given by Sir John Lubbock, we have the following facts and the corresponding explanation of them: "Wind-fertilized flowers, as a rule, have no color, emit no scent and produce no honey, and are regular in form. Color, scent, and honey are the three characteristics by which insects are attracted to flowers. Again, as a rule wind-fertilized flowers produce much more pollen than those which are fertilized by insects. This is necessary, because it is obvious that the chances against any given pollen grain reaching the stigma are much greater in the one case than in the other. Every one has observed the showers of yellow pollen produced by the Scotch fir. Again, it is an advantage to wind-fertilized plants to flower early in the spring before the leaves are out, because the latter would catch much of the pollen, and thus interfere with its access to the stigma. Again, in these plants the pollen is less adherent, so that it can be easily blown away by the wind, which would be a disad-

vantage in most plants which are fertilized by insects. Again, such flowers generally have the stigma more or less branched, or hairy, which evidently must tend to increase their chances of catching the pollen."[1] There is here a structural adaptation of these plants to the circumstances designed to explain them, so that the consequent self-consistent system thus formed carries with it the weight of conviction.

There are some explanations which do not perfectly correspond to reality, and yet, when their nature is known, they may be profitably used, not to represent reality, but to assist the mind by an *approximate* representation to better appreciate the facts as they really are related one to another. These so-called "fictions" are useful, especially in mathematics. We suppose, for instance, inscribed and circumscribed polygons of a circle, with ever-increasing number of sides, gradually approaching and becoming coincident finally with the curve itself. This latter we know to be impossible, and yet we may treat that which happens only approximately as though really happening, merely as an aid to the imagination; and a fiction, if always so understood, may thus prove helpful in the representation of reality more clearly to our minds.

4. The hypothesis, moreover, should involve no contradiction. This is clearly a requirement that is deductive rather than inductive, depending upon the fundamental principle of contradiction lying at the basis of the deductive system of logic.

[1] Lubbock, *Scientific Lectures*, pp. 9, 10.

5. The hypothesis should be as simple as possible. No involved explanation that mystifies rather than clears the difficulties presented can rank as a true hypothesis. *Simplex veri sigillum.* This requirement, of course, cannot in all cases be strictly complied with; for the phenomena to be explained may present such a degree of complexity that a simple hypothesis would be altogether out of the question. For instance, the hypothesis of a substance filling the universe, and pervading all particles of matter, however solid and closely knit together, a substance itself more solid than steel, and more elastic as well, such a supposition seems not only too involved, but also even to belie the ordinary judgments of common sense. And yet this undulatory hypothesis is more and more confirmed by every advance of science in the knowledge of the phenomena of light and heat.

It sometimes happens that the very failure of an hypothesis forms a substantial contribution to the progress of thought, leading to the readjustment of a received theory, or stimulating research in order to discover the true in place of the false hypothesis. As Mr. Tait says: "We all know that if there had not been a pursuit after the philosopher's stone, chemistry could not yet have been anything like the gigantic science it now is. In the same way we can say, that modern physics could not yet have covered the ground it now occupies had it not been for this experimental seeking for the so-called perpetual motion, and the consequent establishment of a definite and scientifically useful negative."[1] The cir-

[1] Tait, *Recent Advances in Physical Science*, p. 69.

cular theory of the orbits of the planets, while incorrect, yet made the transition easier from the hypothesis of circular motion to that of motion in an elliptical orbit, which is the true theory. It often happens that an hypothesis may not be wholly wrong but may need correction, and this is often provided for, not by a total rejection of the hypothesis in question, but by supplementing it by so-called subsidiary hypotheses.

As to the tests of a correct hypothesis in addition to the fulfilment of the requirements already mentioned, Dr. Whewell has especially emphasized the importance of what he has styled a "Consilience of Inductions." An hypothesis receives a confirmatory strengthening of its validity, when it enables us to explain and determine cases not only of the same kind as the phenomena out of which the hypothesis itself has developed, but cases which arise in a sphere entirely different from that which gave material originally for the formation of the hypothesis. An hypothesis that can thus be carried into new territory as an effective instrument of research, is thereby doubly accredited. As Dr. Whewell remarks: "Accordingly the cases in which inductions from classes of parts altogether different have thus *jumped together*, belong only to the best established theories which the history of science contains. And as I shall have occasion to refer to this peculiar feature in their evidence, I will take the liberty of describing it by a particular phrase; and will term it the *Consilience of Inductions*. It is exemplified principally in some of the

greatest discoveries. Thus it was found by Newton that the doctrine of the attraction of the sun varying according to the inverse square of the distance, which explained Kepler's *Third Law*, of the proportionality of the cubes of the distances to the squares of the periodic times of the planets, explained also his *First* and *Second Laws*, of the elliptical motion of each planet; although no connection of these laws had been visible before. Again, it appeared that the force of universal gravitation, which had been inferred from the *perturbations* of the moon and planets by the sun and by each other, also accounted for the fact, apparently altogether dissimilar and remote, of the *precession of the equinoxes.* Here was a most striking and surprising coincidence which gave to the theory a stamp of truth beyond the power of ingenuity to counterfeit."[1]

When two rival hypotheses can be submitted to the test of an experiment which negatives one and confirms the other, such a testing is called an *experimentum crucis.* The name was first given by Bacon, and has met with universal acceptance in scientific phraseology. A crucial test, as decisive between the emission and the undulatory theory of light, is given in an experiment first tried by Father Grimaldi, a Bolognese monk, in 1665. If a shutter be pierced with a very small hole, and the luminous cone which passes through the orifice be examined, the cone will be found to be much

[1] Whewell, *Novum Organon Renovatum,* Bk. II. Ch. V. Art. 110.

less acute than would be expected, considering only the rectilinear transmission of the rays, as according to the emission theory. If there be interposed in the path of the luminous ray a second shutter, pierced with a hole also, it will be noticed that the rays of the second cone are even more divergent than those of the first. If the image of the orifice be received upon a screen, a white circle is seen surrounded by a dark ring, next a white ring, even more brilliant than the central portion, then a second dark ring, and finally another very faint white ring. If in the shutter with which the experiment is made, two very small holes are pierced at a distance from each other of one or two millimetres, and the two images received upon a screen in such a manner that they overlap each other, it is found that in the cuticular segment formed by the overlapping of the images, the circles are more obscure than in the part where they are separated. Thus by adding light to light darkness is produced.[1] These phenomena are now known to be consistent only with the undulatory theory, and directly in contradiction to the emission hypothesis.

M. Romanes performed several experiments upon bees which had the force of crucial tests of two opposed hypotheses, one, that bees possess a general sense of direction, irrespective of any special knowledge of their particular surroundings; the other, that they are guided in their flight by a knowledge of the localities which they have been wont to frequent. M. Romanes took a score of bees in a box out

[1] Saigey, *The Unity of Natural Phenomena*, p. 66.

to sea, where there could be no landmarks to guide the insects home. None of them returned home. Then he liberated a second lot of bees on the sea-shore, and, none of these returning, he liberated another lot on the lawn between the shore and the house. None of these returned, although the distance from the lawn to the hive was not more than two hundred yards. Lastly, he liberated bees in different parts of the flower-garden on either side of the house, and these at once returned to the hive; and with repetition of the experiment, a similar result, even arriving at the hive before he himself had time to run from the place where he had liberated them to the hive. As the garden was a large one, many of them had to fly a greater distance, in order to reach the hive, than those liberated on the front lawn. Their uniform success, therefore, in finding their way home so immediately was no doubt due to their special knowledge of the flower-garden, and not to any general sense of direction.[1]

The hypothesis that leads to verification by experiment represents true scientific procedure, and that which has actually been the most effective instrument of research in all the various spheres of human investigation. The old controversy between Mill and Whewell admits of a ready adjustment in this regard. Whewell emphasized discovery as the heart of the system of induction, leading to the framing of hypotheses whose chief test was not

[1] Lubbock, *On the Senses, Instincts, and Intelligence of Animals*, pp. 269, 270.

experimental so much as the capability of accounting for the given phenomena. Mill, on the other hand, insisted that logic was essentially proof, and not discovery. He, accordingly, emphasized the experimental testing by means of his several methods, as being the all-important part of the inductive method. He had little concern for the origin of the suggestions as to the most likely causal elements in the midst of a complex phenomenon. The primary function of logic, according to him, is merely to prove or disprove. The ideas of Whewell and Mill are not necessarily contradictory; they can be regarded as mutually supplementary, which gives us a true account of the ideal logical method, where hypothesis suggests the line of experiment, and experiment in turn confirms hypothesis. In such a method, as can be seen in the illustration given, there is a blending of deductive and inductive reasoning, which is the general characteristic of all actual processes of thought. As Sigwart has so admirably put it: "Without quickness of combination, by which we can call up a number of possible analogies and apply them to the unexplained case; without a happy power of divination which is guided by unanalyzable associations to discover that analogy which embraces most aspects of the event; finally, without imagination to construct connections for which the only ground may be a hidden similarity, our thoughts, if compelled to proceed strictly according to method, would frequently be condemned, by the impossibility of discovering in this way a sufficiently grounded con-

nection, to complete stagnation. But the fact is in no way contrary to the nature of induction; it is a necessary consequence of it. We cannot even begin the process of inference without making general assumptions; and the general proposition which we get by summing up a number of instances is really a hypothesis, to which, it is true, we are led clearly and certainly in this case. But between these most general presuppositions, upon which all induction is grounded, and the simplest cases to which they can be applied, there is a wide region within which the hypotheses which are always necessary for induction can only be formed tentatively, in order to give some definite direction to investigation, to serve in our analysis of phenomena into their elements as a means of breaking up complete phenomena on certain lines, and to invent the experiments which will make it possible to confirm or refute an opinion."[1]

[1] Sigwart, *Logic*, Vol. II. p. 423.

## CHAPTER XIV

### ANALOGY

It often happens that the cause of a phenomenon is disclosed by the fact that the cause of a similar phenomenon is known, and the inference then follows that the similar phenomena have similar causes. Such a process of inference is determination by analogy. Analogy, considered in its relation to the inductive processes, occupies a twofold position. In the first place, when a complex phenomenon is given, as preliminary to the formation of any hypothesis, as to the probable cause which will, in turn, lead to experimental determination by one of the inductive methods, the mind instinctively examines, with sweeping glance, every detail of the phenomenon for the purpose of discovering some familiar features that may prove suggestive of known relations and functions occurring in other spheres. Analogical suggestion, therefore, initiates every inductive inquiry.

In the second place, in every inductive generalization there is an extension of the known into unknown regions, by virtue of the principle of analogy expressed in what we may style its limiting case. For instance, when we have examined

a number of $A$'s and find them always characterized by the mark $B$, and then by generalization rise to the proposition, All $A$'s are $B$, we do so by reason of postulating an analogy between all the individual $A$'s of so strictly an accurate nature, that it amounts to essential identity. I have therefore called this the limiting case of analogy; and this resemblance of particulars is the ground of all universals whereby they manifest an identity in the midst of differences. We are therefore justified in affirming that all inductive generalizations present an aspect of analogical inference.

Analogy, considered as a mental process, is grounded in the law of similarity. This tendency of noting resemblance makes possible the extension of knowledge. The formation of our concepts is, in the main, an analogical procedure; just as the generalization of an universal depends upon our discrimination of the elements which are similar from those which are different. While analogy thus functions in all the logical processes of thought, it is used in a more restricted sense to indicate that mode of inference especially which proceeds from a number of observed characteristics that are similar, to others which are thereby judged to be similar also. This method is very potent as an instrument of discovery. In 1845, Faraday discovered the magnetic rotary polarization of light; by analogical reasoning, Waitmann in the following year inferred that a similar result would be attained with a beam of heat, which was afterwards experimentally verified. The so-called "natural kinds"

furnish manifold illustrations of conclusive analogies. They possess numerous properties, some of them known and others unknown. Through large groups of them are found similar characteristics side by side with manifest differences, and yet the similarities are so striking that often, when new properties are discovered in certain members of the group, there seems to be ground for inferring their existence in other members of the group also. Certain properties known to exist in potassium and sodium were inferred to be present in rubidium and cæsium; the carbonates of sodium and potassium are not decomposed by a red heat, and it was inferred that the same would prove true of the carbonates of rubidium and cæsium; and such proved to be the case. Some of the statements which are true of chlorine are found to be true of bromine and iodine. Mr. Gore, having found the molecular change in antimony electro-deposited from its chloride, he inferred and discovered the same in that deposited from its bromide and iodide. Sir Humphry Davy, having discovered that potassium might be isolated by means of electrolysis, immediately inferred and proceeded to prove by experiment that it would be possible also to isolate sodium and other substances of analogous properties.[1]

The principle of analogy lies at the basis of all classification, the separating and grouping together in appropriate divisions individuals which possess certain salient attributes in common.

Professor Jevons' definition of classification em-

[1] Gore, *The Art of Scientific Discovery*, p. 522.

bodies at the same time a full statement of its exact logical significance as an instrument of research, and therefore I give it in full: "By the classification of any series of objects, is meant the actual or ideal arrangement together of those which are alike and the separation of those which are unlike, the purpose of this arrangement being, primarily, to disclose the correlations or laws of union of properties and circumstances, and secondarily, to facilitate the operations of the mind in clearly conceiving and retaining in the memory the characters of the object in question."[1] In describing the purpose of classification, the latter clause is more a psychological desideratum than logical; the former specification contains its logical purpose; namely, to disclose the correlations or laws of union of properties and circumstances. This may be illustrated in the grouping together of potassium, sodium, cæsium, rubidium, and lithium, and calling them the alkaline metals. This was done by virtue of the common characteristics in the midst of their individual peculiarities; namely, they all combine very energetically with oxygen to decompose water at all temperatures, and form strongly basic oxides, which are highly soluble in water, yielding powerful caustic and alkaline hydrates from which water cannot be expelled by heat; their carbonates are also soluble in water, and each metal forms only one chloride. The manifest advantage of classifying these metals together lies in its suggestive capacity, as we have

[1] Jevons, *Principles of Science*, p. 677.

already noted in illustrations above given. So many observed similarities suggest inferences by analogy; when, for instance, a new property is discovered in any one or two of the metals of this class, the idea immediately suggests itself that the same property may possibly extend over all the metals of the same class. Not only is such an idea suggested, but along with it there exists an antecedent probability respecting its solution in accordance with the suggestion which analogy starts.

An excellent illustration of the practical results attained through a scientific use of classification is found in Mr. Lockyer's researches on the sun.[1] As a guide as to what elements to look for in the sun's photosphere, he prepared a classification of elements according as they had or had not been traced in the sun, together with a detailed statement of the chemical nature of each element. He was then able to observe that the elements found in the sun were, for the most part, those forming stable compounds with oxygen. He then inferred that the other elements which were known to form stable compounds with oxygen would, in all probability, be found present in the sun. Starting upon this suggested track, he succeeded in discovering five such metals.

Analogical inference carries special weight when it is based upon the principle of teleology; that is, when any observed phenomena seem to possess structural contrivances adapted to ends, in some degree, at least, similar to human contrivances

[1] Quoted by Jevons in *Principles of Science*, p. 676.

designed to produce certain proposed ends. When this similarity is apparent, it suggests the possibility that an observed contrivance in nature may subserve ends beyond the possibility of observation, and which, therefore, may be inferred really to exist. We have seen that the ground of all inference lies in the representation of any given phenomena of consciousness as cohering in one system, which comprehends the several parts in a common unity of such a nature that, knowing some of the parts and their relations, we infer the character and function of other parts not known, and yet which that already known necessitates. And among the many kinds of relation that may obtain between part and part, or part and whole, the teleological is a very common one, and, moreover, by its nature necessitates certain consequences that lie beyond the sphere of observation, and yet, nevertheless, may very properly be supplied by inference. In other words, the causal connections in a system are not merely those of an efficient or a formal cause; they may, with a like force and suggestiveness, be considered in the light of a final cause; that is, the presence of means adapted to certain ends, or of organs adapted to certain necessary functions, or of contrivances of a mechanical nature as though designed for a specific purpose.

Janet has specially emphasized the importance and prevalence of this kind of inference, and, as an illustration of the cogency of inference based upon finality, he urges that the certitude which the belief in the intelligence of our fellow-men gives us is

based upon analogical reasoning of this type; and that, moreover, this belief, resting upon such a basis, is one of the strongest beliefs which we possess. He says: "Now, if we ask ourselves why we suppose that other men think, we shall see that it is in virtue of the principle of final causes. In effect, what is it that experience shows in the actions of other men, but a certain number of phenomena co-ordinated in a certain manner, and bound not only together, but also to a future phenomenon more or less remote? Thus when we see a man prepare his food by means of fire, we know that this assemblage of phenomena is connected with the act of taking food; when we see a painter drawing lines on a canvas, we know that these apparently arbitrary acts are connected with the execution of a picture; when we see a deaf mute making signs which we do not understand, we believe that these gestures are connected with a final effect, which is to be understood by him to whom he makes them; in fine, when men speak, we see that the articulations of which a phrase is composed are co-ordinated to each other so as to produce a certain final effect, which is to awaken in us a certain thought and sentiment. Now we cannot see such co-ordinations, whether actual or future, without supposing a certain cause for them; and as we know by internal experience that with ourselves such co-ordinations only take place under the condition that the final effect is previously represented in our consciousness, we suppose the same thing in the case of other men; in a word,

we suppose for them the consciousness of an end, a consciousness reflecting more or less, according as the circumstances more or less resemble those that accompany in ourselves the reflecting consciousness. Thus when we affirm the intelligence of other men, we affirm a truth of indisputable certitude; and yet we only affirm it on the ground of analogy, and of analogy guided by the principle of final causes."[1]

In this illustration of Janet's we have the idea of a system of co-ordinated parts especially prominent; and for a satisfactory account of the relations obtaining in such a system, it will be seen how indispensable it is to postulate the theory of final cause. This mode of inference finds a striking illustration in the famous discovery of Harvey, concerning the circulation of the blood. In the early part of the seventeenth century, while Harvey was his pupil, the celebrated anatomist, Fabricius Aquapendente of Padua, observed that many veins contain valves which lie open as long as the blood is flowing towards the heart. Harvey, learning of this fact, saw in it the suggestion of an adaptation of means to an end; namely, a contrivance so fashioned by nature as to permit the blood to flow always in one direction only, and to prevent its flow in an opposite direction. Observation of other portions of the circulatory mechanism led to a confirmation of the idea, and to the discovery of the circulation of the blood.[2]

[1] Janet, *Final Causes*, pp. 113, 114.
[2] Gore, *Art of Scientific Discovery*, p. 571.

Again, many flint substances have been discovered, as though curiously wrought, with sharp edges and a place as though designed for a handle, with which to wield the stone as a weapon or a tool; it has been inferred from these general characteristics that the stones were so constructed by human effort, and used by human beings for the purposes for which they evidently seem to be adapted. This inference is based upon an analogy between the peculiar shapes of such stones, and known shapes designed and used by man.

This form of analogy has proved specially suggestive in researches regarding plant and animal life. Sir John Lubbock gives the following description of the common white dead-nettle, with the explanation of its functions that is evidently a teleological inference: "The flower consists of a narrow tube, somewhat expanded at the upper end, where the lower lobe of the corolla forms a platform, on each side of which is a small projecting lobe. The upper portion of the corolla is an arched hood, under which lie four anthers in pairs, while between them and projecting somewhat downwards is the pointed pistil. At the lower end, the tube contains honey, and above the honey is a row of hairs almost closing the tube. Now, why has the flower this peculiar form? What regulates the length of the tube? *What is the use of this arch?* What lessons do these lobes teach us? What advantage is the honey to the flower? *Of what use* is the fringe of hairs? Why does the stigma project beyond the anthers? Why is the corolla white, while the rest

of the plant is green? Similar questions may of course be asked with reference to other flowers. At the close of the last century, Conrad Sprengel published a valuable work, in which he pointed out that the forms and colors, the scent, honey, and general structure of flowers, have reference to the visits of insects, which are of importance in transferring the pollen from the stamens to the pistil. Mr. Darwin developed this theory and proved experimentally that the special service which insects perform to flowers, consists not only in transferring the pollen from the stamens to the pistil, but in transferring it from the stamens of one flower to the pistil of another."[1] The line of subsequent observation and experiment was thus originally suggested by the structural appearance of these flowers which seemed formed for some specific end. The questions, once started, — To what end? To what purpose? For what use? — led to the theory of Sprengel and the corroborative experiments of Darwin.

This is further illustrated in some very interesting flower structures, also described by Sir John Lubbock, which indicate peculiar contrivances for the destruction of insects. The peculiarity of formation first suggested some such end as this, which has since been proved by careful observation to be the case. "The first observation on insect-eating flowers was made about the year 1868 by Ellis. He observed that in Dionæa, a North American plant, the leaves have a joint in the

[1] Lubbock, *Scientific Lectures*, pp. 1, 2.

middle, and thus close over, kill, and actually digest any insect which may alight on them. Another case is that of Utricularia, an aquatic species which bears a number of utricles or sacs, which have been supposed to act as floats. Branches, however, which bear no bladders float just as well as the others, and there seems no doubt that their real use is to capture small aquatic animals, which they do in considerable numbers. The bladders, in fact, are on the principle of an eel-trap, having an entrance closed with a flap, which permits an easy entrance, but effectually prevents the unfortunate victim from getting out again. In the genus, Sarracenia, some of the leaves are in the form of a pitcher. They secrete a fluid, and are lined internally with hairs pointing downwards. Up the outside of the pitcher there is a line of honey glands which lure the insects to their destruction. Flies and other insects which fall into this pitcher cannot get out again and are actually digested by the plant."[1]

In the example where the idea of an eel-trap suggested the possible function of the similar structure in the plant, Utricularia, we find one of the most striking illustrations of this mode of analogical inference. It was an easy and natural transition from similarity of structure to similarity of function. To give an idea of the great number of teleological phenomena in the vegetable and animal world, and the wealth of possible suggestion stored away in these various structures, and disclosed by a sagacious analysis, I quote a remark

[1] Lubbock, *Scientific Lectures*, pp. 4, 5.

of Sir John Lubbock's in commenting upon the variation of color and markings of caterpillars: "I should produce an impression very different from that which I wish to convey, were I to lead you to suppose that all these varieties have been explained or are understood. Far from it; they still offer a large field for study; nevertheless, I venture to think the evidence now brought forward, however imperfectly, is at least sufficient to justify the conclusion that there is not a hair or a line, not a spot or a color, for which there is not a reason, — which has not a purpose or a meaning in the economy of nature."[1]

An illustration given by Darwin shows this mode of inference applied to the sphere of animal life also. He says: "The great size of the bones of the megatherioid animals was a complete puzzle to naturalists until Professor Owen lately solved the problem with remarkable ingenuity. The teeth indicate, by their simple structure, that these megatherioid animals lived on vegetable food, and probably on the leaves and small twigs of trees; their ponderous forms and great, strong, curved claws *seem so little adapted* for locomotion that some eminent naturalists have actually believed that, like the sloths, to which they are intimately related, they subsisted by climbing back downwards on trees, and feeding on the leaves. It was a bold, not to say preposterous, idea, to conceive even antediluvian trees with branches strong enough to bear animals as large as elephants. Professor Owen,

[1] Lubbock, *Scientific Lectures*, pp. 66, 67.

with far more probability, believes that, instead of climbing on the trees, they pulled the branches down to them, and tore up the smaller ones by the roots, and so fed on the leaves. The colossal breadth and weight of their hinder quarters, which can hardly be imagined without having been seen, become, on this view, of obvious service, instead of being an encumbrance: their apparent clumsiness disappears. With their great tails and their huge heels firmly fixed like a tripod on the ground, they could freely exert the full force of their most powerful arms and great claws. Strongly rooted, indeed, must have been that tree which could have resisted such force! The Mylodon, moreover, was furnished with a long extensile tongue like that of the giraffe, which, by one of those beautiful provisions of nature, thus reaches, with the aid of its long neck, its leafy food."[1] Throughout we observe analogical inference based upon these teleological marks, and furnishing a basis for a satisfactory hypothesis.

We see what a wide field thus opens in the region of biology alone for the discovery of resemblances leading to the appreciation of the fuller teleological significance of plant and animal life.

In the illustrations given, both of the teleological and other forms of analogy, we notice that its chief logical function is that of suggestion of some hypothesis which may or may not be afterwards confirmed by subsequent experiment. Some of the most important discoveries of science have arisen from analogical suggestions. Sir John Herschel

[1] Darwin, *Voyage of a Naturalist*, pp. 106, 107.

was led by observed analogies to predict certain phenomena afterwards verified experimentally by Faraday. Herschel had noticed that a screw-like form, known as helicoidal dissymmetry, was observed in three cases, namely, in electrical helices, plagihedral quartz crystals (that is, crystals having an oblique spiral arrangement of planes), and the rotation of the plane of polarization of light. As Herschel himself said: "I reasoned thus: Here are three phenomena agreeing in a *very strange peculiarity.* Probably this peculiarity is a connecting link, physically speaking, among them. Now, in the case of the crystals and the light, this probability has been turned into certainty by my own experiments. Therefore, induction led me to conclude that a similar connection exists, and must turn up, somehow or other, between the electric current and polarized light, and that the plane of polarization would be deflected by magneto-electricity." Herschel thus anticipated Faraday's experimental discovery of the influence of magnetic strain upon polarized light.[1]

Another important discovery — the germ-theory of epidemic disease — was first suggested by an analogy. In the theory, as expressed by Kircher, and favored by Linnæus, and afterwards supported by Sir Henry Holland, its special strength, according to Professor Tyndall, "consisted in the perfect parallelism of the phenomena of contagious disease with those of life. As a planted acorn gives birth to an oak competent to produce a whole crop of

[1] Jevons, *Principles of Science*, p. 630.

acorns, each gifted with the power of reproducing the parent tree, and as thus from a single seedling a whole forest may spring, so, it is contended, these epidemic diseases literally plant their seeds, grow and shake abroad new germs, which, meeting in the human body their proper food and temperature, finally take possession of whole populations."[1]

The theory of evolution was first suggested to Mr. Darwin by the analogous phenomena observed in artificial selection and breeding. The transition to natural selection was easily made, especially as, on reading Malthus, *On Population*, he conceived the idea of a struggle for existence as the inevitable result of the rapid increase of organic beings. This idea necessitated the natural selection, which he needed to account for results similar to the artificial selection, and thus his theory grew out of an analogy as its beginning. Moreover, in the development of the theory in its manifold details, other analogies proved also suggestive. For instance, there is the supposed analogy between the growth of a species and the growth of an individual. It supposes, for example, as Professor Clifford has put it, "that the race of crabs has gone through much the same sort of changes as every crab goes through now, in the course of its formation in the egg, — changes represented by its pristine shape utterly unlike what it afterwards attains, and by its gradual metamorphosis and formation of shell and claws."[2]

[1] Tyndall, *Fragments of Science*, p. 287.
[2] Clifford, *Lectures and Essays*, p. 86.

The germ-theory of putrefaction, first suggested by Schwann, received confirmation through certain resemblances noted by Professor Lister between fermentation and putrefaction. In his Introductory Lecture before the University of Edinburgh, Professor Lister called attention to the fact that fermentation and putrefaction present a very striking parallel. In each a stable compound — sugar in one case, albumen in the other — undergoes extraordinary chemical changes under the influence of an excessively minute quantity of a substance which, regarded chemically, would be considered inert. It was pointed out, also, by Professor Lister, in this connection, that, as was well known, one of the chief peculiarities of living organisms, is that they possess extraordinary powers of effecting chemical changes in materials in their vicinity out of all proportion to their energy as mere chemical compounds. Such being the facts in the case, and, moreover, the fermentation of sugar being generally allowed to be occasioned by the presence of living organisms, Professor Lister's inference was that putrefaction was due to an analogous agency.[1]

A discovery in quite a different sphere, that of mathematics, leading to the branch of analytical geometry, was first suggested to Descartes through observing the resemblances existing between geometry and algebra. In a similar manner, Boole was led by the resemblances noted between algebra and logic, to give expression to the same in a sys-

[1] Tyndall, *Fragments of Science*, pp. 300–302.

tem which he called the laws of thought, and which has become the basis of a general or symbolic logic.

While there are thus unquestionable evidences of the value of analogy as a form of inference, there are also cases of false analogy unfortunately so numerous as to discredit the process wholly in some quarters. It will be well, therefore, to indicate some of the requirements of true analogy: —

1. In the first place the resemblance must be a preponderating one; that is, the phenomena compared must show a more striking agreement than difference. Some writers have balanced agreement against difference upon a purely numerical basis of comparison, forming what may be called an analogical ratio, with points of similarity forming the numerator, and the points both of similarity and difference, plus the unknown, that is, the total number, forming the denominator. Such a representation of the force of an analogy is given by Mill, Bain, and others. I think, however, that this representation is apt to be misleading in producing the impression that the mere number of points of agreement, irrespective of their significance, is the chief feature of analogy. Whereas it is the weight of the agreeing attributes, and not the number, that counts. As has been before said, in analogy we weigh instances, and do not count them. The analogical ratio expressed numerically, as above, is really equivalent to the ratio of probability which will be described in the following chapter. I have therefore changed the usual wording of this requirement, so that it reads, the resemblances must be more striking than

the differences. This provides for cases when perhaps a few points of resemblance will be of such a nature as to outweigh many points of difference in the total estimate.

This requirement also excludes all fanciful analogies and all resemblances resting upon a figurative rather than a real basis. For instance, the advocates of annual Parliaments in the time of the Commonwealth, urged their case on the analogical ground that a body politic is similar to a living body and that serpents annually cast their skin, which, being no doubt for a beneficial purpose, might well be imitated.

2. In noting the points of resemblance between two phenomena, all circumstances which are known to be effects of one cause must therefore be regarded not as many, but as one. For instance, two chemical oxides may be compared; the effects common to each may be due to the presence of the oxygen which each contains and therefore must not be regarded in the light of independent marks of similarity.

3. If we infer by analogy that a substance possesses a certain property which we know is incompatible with some one or other known properties of the substance, the analogy is at once discredited. We may infer that the moon is inhabited, by virtue of the many points of resemblance between the moon and the earth. However, the fact that the moon has no atmosphere necessary to sustain life, at once makes such an argument based upon analogy wholly out of the question.

4. There are certain special requirements referring to that particular form of analogy which is based upon teleological considerations. They are as follows: —

*a.* This principle must never be used as an argument against an observed fact, or an established law of nature. While this precaution is not necessary at the present time, in scientific circles at least, still there was a time when its counsel was sorely needed. When in astronomy it was proved that there were suns gravitating around other suns, without our solar system, this was objected to upon the following ground, as given by one Nicholas Fuss, a celebrated astronomer, at the end of the eighteenth century: "What is the good of some luminous bodies revolving round others? The sun is the only source whence the planets derive light and heat. Were their entire systems of suns controlled by other suns, their neighborhood and their motions would be *objectless,* their rays *useless.* The suns have no need to borrow from strange bodies what they themselves have received as their own. If the secondary stars are luminous bodies, *what is the end* of their motives?"

There is, moreover, another abuse of the principle of final causes, which has also historic interest rather than any present pertinence; namely, opposing certain false teleological ideas to established discoveries or inventions, with a mistaken zeal, in defence of a Divine Providence. For instance, at the time of Jenner's great discovery, an English physician, Dr. Rowley, said of small-pox: "It is a

malady imposed by the decree of heaven, and vaccination is an audacious and sacrilegious violation of our holy religion. The designs of these vaccinators appear to defy heaven itself, and the very will of God." The introduction of winnowing machines into Scotland met with bitter opposition on the ground that the winds were the work of God, and that the wind thus artificially raised was a veritable "devil's wind," as they were wont to call it. Sir Walter Scott, in *Old Mortality*, has the old Mause say to her mistress: "Your ladyship and the steward are wishing Cuddie to use a new machine to winnow the corn. This machine opposes the designs of Providence, by furnishing wind for your special use, and by human means, in place of asking it by prayer, and waiting with patience till Providence itself sends it."

*b.* Final causes should never be employed to explain phenomena which do not exist. As M. Florens has said: "We must proceed not from final causes to facts, but from facts to final causes; that is, we should not superimpose final causes upon phenomena. We must see them in phenomena themselves, and we must not arbitrarily project a teleological idea, purely subjective, upon an objective ground. Thus in ancient times, Hippocrates is said to 'have admired the skill with which the auricles of the heart have been made *to blow the air into the heart.*'"

*c.* We must distinguish accidental from essential marks of finality, and not be led into fanciful or far-fetched analogies. Voltaire has expressed such

a defect when in satire he made that famous remark, "Noses are made in order to bear spectacles."

Bernardin de Saint-Pierre says: "Dogs are usually of two opposite colors, the one light, the other dark, in order that whenever they may be in the house, they may be distinguished from the furniture, with the color of which they might be confounded. . . . Wherever fleas are they jump on white colors. This instinct has been given them, that we may the more easily catch them." And again the same writer says: "The melon has been divided into sections by nature, for family eating." [1] All such grotesque inferences will give an idea of how readily the imagination will run riot if allowed to remain uncurbed by the reason.

5. Analogy should never be regarded as having more weight than that of extremely high probability, even in cases seemingly most conclusive. Its true function is suggestive, leading to hypothesis and experiment, and it needs this supplementary proof. It was an inference based on analogy, for instance, which suggested the probability that the Binomial Law, having proved to be valid as regards the second, third, and fourth powers, might also be extended to the fifth, and so on to the other powers indefinitely. This suggestion offered no real basis, however, upon which the Binomial Theorem could rest; it needed mathematical demonstration to confirm and generalize its expression in the special cases already experimentally tested, so as to cover

[1] The illustrations upon the abuse of final causes I have taken from Janet's admirable chapter, — Chapter VIII. of Appendix.

all possible exponents, both positive and negative, fractional and integral.

So also the discovery of the circulation of the blood was first suggested to Harvey, as has been said, by analogical considerations upon observed teleological phenomena. Harvey, however, was not content with this suggestion merely. He was led to experiment upon the veins and arteries; he tied an artery and vein, and carefully observed the mechanical effects upon the two sides of the tied parts. Experiments of this nature, with close observation and study, were kept up most diligently, and with rare perseverance, for *nineteen* years, before he had traced the entire course of the blood through all parts of the human body, and, in a manner wholly satisfactory to himself, verified the first statement of this theory.

## CHAPTER XV

### Probability

There are certain phenomena of such a nature that their antecedents, being extremely complex, cannot be adequately comprehended by observation, however searching it may be; nor can they be subjected to any analysis that will disclose the causal elements to which the effect in question is due. Moreover, with seemingly the same antecedents, the event sometimes happens, and sometimes does not; and even with antecedents associated with an event as cause and effect respectively, nevertheless the event does not occur as we should naturally expect, while with antecedents associated with the contradiction of the event as cause and effect respectively, we find the occurrence of the event quite contrary to what we should naturally expect. The evidence of a constant connection between antecedent and consequent, that we have found in so many cases which we have examined, is here wholly lacking. Regularity has been replaced by irregularity respecting such phenomena. For instance, I throw dice repeatedly; the antecedent shaking of the box, and tossing the dice upon the table, is about the same each time, at least the difference can-

not be determined, and yet the results vary with each successive throw. The causal determination in each case is so complex as to be beyond computation; the initial position of the dice, the force of their ejection from the box, the height of the box above the table when they leave it, the inequalities of the table itself, a variation between the physical and geometrical centres of gravity of the dice, etc., all these make the antecedent so complex that a slight variation in any one of these conditions will affect the result. We find, therefore, double sixes at one time, a three and four at another, and so on indefinitely.

Or, again, it sometimes happens that with perfect sanitary conditions a contagious disease will appear, that has always been regarded, and that correctly, as due to imperfect sanitation; or, an entire disregard of sanitary requirements and of all the laws of health may yet give rise to no disease of special moment. Certain conditions of temperature, atmospheric pressure, velocity and direction of the wind, may one day bring storm and rain, and as far as observation can detect, similar conditions may again bring fair weather. So, also, the rise and fall in stock and money markets is extremely susceptible to the varying conditions of indefinitely complex forces wholly beyond all powers of determination or of prediction. Such phenomena present a problem which the methods of inductive inquiry cannot deal with. Observation is not far-reaching enough to provide the data for the solution of the problem, and, even if it were,

our methods of computation and determination are not sufficiently adequate to solve problems of so many terms and of so complex a nature.

The experimental methods are designed to test causes suggested by analogy, or a mental analysis; but in such phenomena as these, the problem is not simply to find a causal connection. The causal connection may be established beyond all reasonable doubt, and yet the cause obtains in the midst of so complex a setting that the problem is really this,—to determine whether a cause, whose exact nature may be known or unknown, will prove operative or inoperative. The cause may be always present and even its exact nature may be known, and yet the complex circumstances attending it may be of such a character that one alone, or two or more combining, may neutralize the operation of the cause, and on the other hand a slight variation of the combined circumstances may promote and even accelerate the operation of the cause in question. The problem then is to determine how often the event happens, and how often it fails of happening, the complex and indeterminate antecedent being present in all the instances examined.

When we begin to count instances, we are reminded that we must be in the near neighborhood of the sphere of enumerative induction. Enumerative induction, it will be remembered, treats instances by noting the number of observed coincident happenings of the antecedent and consequent under investigation, no attempt being made to analyze

their respective contents, or to determine a causal connection more definitely by means of any one or more of the inductive methods of research and verification. The result of such an investigation may be formulated in a proposition of the form, Every $A$ is $B$. This, strictly interpreted, has the force of, Every $A$ that has been observed is $B$. The enumeration of the kind of instances which we are discussing in this chapter, however, differs from this in that the observation leads to a twofold result, — a set of instances in which it is observed that the $A$'s *are* $B$'s also, another set, however, in which the $A$'s *are not* $B$'s. These instances are of such a nature that the observed $A$ is an antecedent so extremely complex that the element within it, which is a cause capable of producing $B$, may either be absent without producing an appreciable change in the general nature of $A$, or, being present, may be neutralized by some other element of $A$ itself. The result gives a basis for a probable inference only; and the nature of that inference will depend upon the preponderance of the observed happening, or of the failure of the event under investigation.

The probability attached to such an inference, however, is different from the probability which characterizes the nature of enumerative induction. In the latter, when the observation has been widely extended and no exceptions noted, it is usual to say the result expressed in the proposition, Every $A$ is $B$, has the force of a high degree of probability. In the instances, however, whose investigation

shows the result that some $A$'s are $B$'s, and some not, and yet where the former, for instance, far outnumber the latter cases, then it may be inferred that the $A$'s which in future we may meet with will probably be $B$'s; and the degree of probability expressed in such a proposition is commensurate with the preponderance of the number of observed affirmative instances over the negative. Here the probability refers to the validity of an inference concerning certain *particular* instances, be they many or be they few, which lie beyond the sphere of our present knowledge; in enumerative induction, the probability is attached to the *universality* of the proposition affirmed as a result of observation that has not so far detected an exception. In the former case, the question of the universality of the result is conclusively answered, and that in the negative; there can be no universal proposition possible, as some instances give $A$ and $B$ together, others give $A$ with the absence of $B$; and the question of probability that here arises, therefore, refers to individual cases not yet examined, as to whether they severally will more likely correspond to the set of affirmative, or to that of the negative instances already noted.

The comparison of the number of happenings with that of the failures of an event affords a basis for three kinds of inference, all of them in the sphere of probability.

1. We find in such a comparison a basis for the calculation of the probability of a particular event happening, in case there is a repetition of the cir-

cumstances which, in former cases, have sometimes produced the event, and sometimes have failed to produce it. If, according to former observation, the event has happened, let us say, seven times, and failed three, the probability, expressed numerically, of its happening again is $\frac{7}{10}$. The rule is, to express the probability of an event, take as numerator the number of times which the event has been observed to occur, and as denominator the total number observed, both of happening and failure; the fraction thus expressed will represent the probability of the event happening. The counter-probability may be represented by the number of observed failures of the event divided by the total number of cases observed. The counter-probability, plus the probability, evidently is equal to unity. If, therefore, the probability is unity, the counter-probability will equal zero; that is, the probability in that case has merged into certainty. Zero, therefore, represents absolute impossibility. All fractions between the limits zero and one represent varying degrees of probability from impossibility at one extreme to certainty at the other.

Not only may there be this inductive basis for the calculation of probability, arising from actually observed instances; there may be also a deductive calculation of probability based upon the known structure or nature of the phenomena themselves in advance of any observation as to their actual behavior. For instance, we say the probability of a penny turning up heads is $\frac{1}{2}$. Knowing the form of the penny and that there are but two

possibilities, heads or tails, and there being no reason why one should more likely turn up than the other, we say there is one chance favorable to heads as over against the two chances which represent the total number of possibilities under the existing circumstances. With a die, in the form of a perfect cube, we say there is one chance of its turning up the face marked 1, as over against the six chances represented by the six faces, the total number; here the probability is $\frac{1}{6}$. Thus the basis for the calculation of probability may be a theoretical as well as an empirical one.

In the estimate of the probability of an event in the actual conduct of affairs, we seldom express that probability numerically. I would say that we express a degree of probability adverbially rather than numerically; that is, we say an event is *quite* probable, or it is *very* probable, or it is *extremely* probable. The fact is that, as regards most phenomena, we do not keep an exact or even approximate memorandum of the number of happenings compared with that of the failures. We rather classify our observations in terms of more or less. For instance, certain circumstances we observe produce about as many failures as happenings of an event; other circumstances produce far more happenings than failures; others far less, and so on. Consequently we receive certain psychological impressions of varying degrees of intensity according to the preponderance of happening over failure, or *vice versa;* this impression becomes the basis for estimating the probability in question, and the degree of that

probability is commensurate with the intensity of the original psychological impression arising from concepts of more or of less. In such a sphere, however, as that devoted to the interests of betting, gambling, pool-selling, book-making, etc., probabilities are estimated according to observations and theoretical considerations, whose conditions are expressed numerically; and the amount risked in each case is strictly estimated according to the exact ratio of probability to counter-probability under the existing circumstances.

The estimation of probability in terms of a greater or less degree is, however, more usual, and applicable to the conduct of human life generally. It has special force and utility as a mode of inference, when the observed instances so far outnumber the exceptions as to create an impression of such a high degree of probability as to approximate practical if not theoretical certainty. For instance, it has been noted over a wide field of observation, that a second attack of scarlet fever is extremely rare. Exceptions have occurred and, therefore, by enumerative induction it is impossible to generalize the universal proposition that a second attack will never occur. It is, however, possible to assert with somewhat positive assurance that it is highly probable that a person will be exempt from a second attack.

Or, you hear that a person, whose name is unknown to you, has met with an accident in the city of New York, resulting fatally. You are not alarmed, and perhaps the possibility does not even

suggest itself to you, that the unknown person may prove to be a member of your own family, or a friend who at the time is known to be in New York. The probability against such a suggestion is so large as to preclude even the thought of it. Suppose, however, the accident occurred at one of the suburban stations. Your knowledge that your friend rides on one of the suburban trains each day to and from town, may be the ground of some anxiety, because in this case the range of possibilities is materially narrowed. Suppose, moreover, that the station where the accident occurred is at the village where your friend resides, your anxiety receives an additional increment; and, again, suppose it is at the hour at which your friend ordinarily reaches this station, there is then increased apprehension on his account. Thus, as further knowledge limits the number of total possible cases, the denominator of the probability fraction is continually decreasing, and therefore the probability itself continually increases, until it has developed from a fraction of insignificant proportions to one which is suggestive of great anxiety and suspense.

2. The comparison of failure and happening of events based upon observation, or theoretical considerations of structure and nature, leads also to inferences concerning large numbers of instances considered together. If a memorandum is kept of the number of times an event has happened, and the number of times it has failed, and the total number of instances examined be sufficiently great, then the resulting ratio of favorable instances to

the total number will be found approximately repeated, if a second set of an equal number of instances be likewise examined. There is a law of tendency whereby nature seems to repeat herself even when the attendant circumstances of an event are most complex, and beyond all powers of accurate determination. As the result of observations extending over thousands and thousands of instances, it is affirmed that about one-fourth of the children born in the world die before the age of six years, and about one-half before the age of sixteen. Take a group of ten children, the ratios would perhaps be deviated from very materially; in a group of a hundred the deviation is apt to be less; in a group of a thousand, still less; and in a group of one hundred thousand, the ratios as above given would be substantially realized. The approximation would be so near that the error would be insignificant as compared with total number of cases. The following law, therefore, expresses this tendency, — that while in a small number of instances there is irregularity in the observed ratio between the number of times a given event has happened and its failures, still in a large number of instances this ratio tends towards a constant limit. This is clearly seen in the pitching of a penny; 10 throws might very possibly result in 7 heads and 3 tails; in 100 throws, however, the ratio expressing the result as to heads and tails observed will be much nearer $\frac{1}{2}$ than in the former case; while if 1000 or 10,000 throws be observed, the result will approximate the ratio $\frac{1}{2}$. The comparison of observed cases

with the number given by the calculation of the probabilities in question has been made by Quetelet, also by Jevons. Their results are most significant and interesting. Quetelet made 4096 drawings from an urn containing 20 black balls and 20 white. Theoretically, he should have drawn as many white as black balls, 2048 each; the actual drawings resulted in 2066 white balls and 2030 black. Jevons made 20,480 throws of a penny; the theoretical result should have been 10,240 heads; the actual result was 10,353 heads.

The tendency towards a constant ratio in aggregates containing a considerable number of instances is strikingly illustrated in the record of baptisms taken from an old parish register in England. The number of male baptisms registered to every 1000 female ran as follows for the respective years from 1821 to 1830: 1048, 1047, 1047, 1041, 1049, 1046, 1047, 1043, 1043, 1034. We see with what surprising accuracy the constant ratio was repeated substantially, year after year. This tendency to approximate a constant ratio is seen even in such indeterminate events as railroad accidents. Here the causes producing the accidents are so numerous, so diverse, so complex and extending over so large an area, as, for example, the whole of the United States, that we should think that the results would exhibit so many variations from any definite ratio as absolutely to elude all attempts at accurate determination. The following figures, however, given by the Interstate Commerce Commission, indicate results wonderfully corresponding for year after year: —

## RAILROAD ACCIDENTS IN THE UNITED STATES

(As reported by the Interstate Commerce Commission, Washington, D.C.)

| Year Ending June 30. | Employés. | | Passengers. | | Other Persons. | | Total. | |
|---|---|---|---|---|---|---|---|---|
| | Killed. | Injured. | Killed. | Injured. | Killed. | Injured. | Killed. | Injured. |
| 1888 . . . . . . . | 2070 | 20,148 | 315 | 2138 | 2897 | 3682 | 5282 | 25,888 |
| 1889 . . . . . . . | 1972 | 20,028 | 310 | 2146 | 3541 | 4135 | 5823 | 26,309 |
| 1890 . . . . . . . | 2451 | 22,396 | 286 | 2425 | 3598 | 4206 | 6335 | 29,027 |
| 1891 . . . . . . . | 2660 | 26,140 | 293 | 2972 | 4076 | 4769 | 7029 | 33,881 |
| 1892 . . . . . . . | 2554 | 28,267 | 376 | 3227 | 4217 | 5158 | 7147 | 36,652 |
| 1893 . . . . . . . | 2727 | 31,729 | 299 | 3229 | 4320 | 5435 | 7346 | 40,393 |

The total number of passengers carried was 593,560,612 in 1893, as against 560,958,211 in 1892, being an increase of 32,602,401. Casualties at stations, highway crossings, and trespassers upon tracks are included in above table under the heading "other persons."

| Kind of Accident. | Employés. | | Passengers. | | Others. | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | Trespassing. | | Not Trespassing. | | Total. | |
| Year ending June 30, 1893. | Killed. | Injured. | Killed. | Injured. | Killed. | Injured. | Killed. | Injured. | Killed. | Injured. |
| Coupling and uncoupling | 433 | 11,227 | ... | ... | ... | ... | ... | ... | ... | ... |
| Falling from trains and engines | 644 | 3,780 | ... | ... | ... | ... | ... | ... | ... | ... |
| Overhead obstructions | 73 | 444 | ... | ... | ... | ... | ... | ... | ... | ... |
| Collisions | 247 | 1,491 | 68 | 772 | 32 | 38 | 14 | 64 | 46 | 102 |
| Derailments | 153 | 867 | 22 | 774 | 25 | 43 | 4 | 42 | 29 | 85 |
| Other train accidents | 125 | 650 | 10 | 157 | 84 | 124 | 7 | 19 | 91 | 143 |
| At highway crossings | 32 | 43 | 2 | 15 | 163 | 179 | 431 | 870 | 594 | 1049 |
| At stations | 117 | 1,258 | 65 | 568 | 379 | 409 | 75 | 143 | 454 | 552 |
| Other causes | 903 | 11,919 | 132 | 943 | 2990 | 3216 | 116 | 288 | 3106 | 3504 |
| Total | 2727 | 31,729 | 299 | 3229 | 3673 | 4009 | 647 | 1426 | 4320 | 5435 |

Train accidents for twenty-one years ending December 31, as computed by the *Railroad Gazette*.

| Kind of Accident. | 1893. | 1892. | 1891. | 1890. | 1889. | 1888. | 1887. | 1886. | 1885. | 1884. | 1883. | 1878–82.* | 1873–7.* |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Collisions | 996 | 1062 | 1137 | 1041 | 749 | 804 | 700 | 501 | 464 | 445 | 630 | 417 | 295 |
| Derailments | 1212 | 1165 | 1204 | 1004 | 759 | 1032 | 705 | 641 | 681 | 681 | 926 | 646 | 709 |
| Other accidents | 99 | 100 | 103 | 101 | 61 | 99 | 86 | 69 | 72 | 65 | 84 | 46 | 61 |
| Total | 2307 | 2327 | 2444 | 2146 | 1569 | 1935 | 1491 | 1211 | 1217 | 1191 | 1640 | 1109 | 1067 |

* Average per year for five years.

An examination of these figures will disclose the fact that there is a striking approximation to an accurately proportionate distribution of the number of accidents, of the killed and of the injured, throughout these several years. It will be noticed, also, that the distribution among employés, passengers, other persons, etc., tends towards a regularity that is remarkable when we consider the extreme complexity of the circumstances that must combine to produce these results. A like regularity seems to pervade every department of life. The total number of crimes is approximately the same, year after year; the annual death-rate, the apportionment of deaths, moreover, to the several diseases as their evident causes, the number of missent letters that reach the Dead-Letter Office at Washington each year, the annual number of suicides, of divorces, all these diverse events indicate a regularity, in the long run, as regards their numerical estimate.

The results which are thus attained regarding aggregates cannot be stated as probable results. If a sufficiently large number of instances are taken, the result will be certain within a very small, and in many cases an insignificant margin. In estimating the probability of a single event, the question is whether it will happen or not happen, and the element of uncertainty is therefore prominent. In dealing with aggregates, however, no such element of uncertainty enters; the question is not whether or not there will be certain results, the question concerns rather the degree of exactness with which the results will approximate a definite ratio. And

the law of tendency is, that the larger the number of instances, the greater will be the approximation to an accurate and definite result.

This is especially illustrated in the numerous insurance companies, whose business is conducted upon the basis of an approximately constant death-rate. For instance, the general procedure is somewhat as follows: Suppose 10,000 persons insure their lives at $1000 per individual, and the annual death-rate observed over a rude extent of territory, and including a very large number of instances, amounts to 200 persons out of 10,000. The losses then to the insurance company will amount annually to $200,000 on such a basis. These losses, distributed among the 10,000 insuring in the company, would amount to $20 apiece. The company, therefore, has a numerical basis for calculating the amount which each person must pay in order to cover the annual losses, and provide an assured revenue for the company.

I have, of course, stated the problem in round numbers, merely to illustrate in general the principle involved; the actual calculation is more complicated, because, in each particular case, the age of the individual and the varying death-rates for different years must be taken into account. The substantial standing of the innumerable insurance companies in our country bears witness to the fact that these enterprises are based upon a practical certainty regarding death-rates when applied to large aggregates. Chance is thus eliminated almost entirely; that which would be a serious risk as regards an in-

dividual is substantially void of all risk when large numbers are concerned.

Moreover, statistics covering different classes are often most valuable in indicating tendencies operative in the classes when compared one with another. According to M. Loua (*Economiste Français*, 1882, Vol. I. p. 179), the following are the figures of the annual mortality in Paris: —

The rich and well-to-do classes, 156 out of every 10,000 inhabitants.

The poor, 285 out of every 10,000 inhabitants.

So also, in England, the average duration of life among the wealthy classes is from 55 to 56 years; for the working classes it falls to 28 years, or even lower.[1] Such comparisons are significant in indicating underlying forces in society that otherwise might be overlooked, or, at least, not adequately appreciated, and which a limited observation could not accurately reveal. Mr. Darwin, after observing and experimenting upon a very large number of plants, found the following figures respecting the relative productivity of cross and spontaneously self-fertilized flowers: As regards the number of seeds per pod yielded by cross and self-fertilized flowers, the ratio was 100 to 41 respectively; the crossed seeds compared with an equal number of the spontaneously self-fertilized seeds were heavier, in the ratio of 100 to 88.[2] The ratios thus disclosed in examining a large number of instances could not have been gained by any experimental method

[1] Gide, *Political Economy*, p. 405.

[2] Darwin, *Cross and Self Fertilization*, p. 165.

adapted for dealing with individual instances. Although the cause is not quantitatively determined, a tendency of a constant nature towards a definite end is clearly indicated.

Race characteristics are often disclosed by comparative statistics, and the presence or absence of moral causes especially are thus revealed which otherwise could not be determined with any considerable degree of definiteness. The following tables will indicate this: —

*Suicides.* — In European cities the number of suicides per 100,000 inhabitants is as follows: Paris, 42; Lyons, 29; St. Petersburg, 7; Moscow, 11; Berlin, 36; Vienna, 28; London, 23; Rome, 8; Milan, 6; Madrid, 3; Genoa, 31; Brussels, 15; Amsterdam, 14; Lisbon, 2; Christiania, 25; Stockholm, 27; Constantinople, 12; Geneva, 11; Dresden, 51. Madrid and Lisbon show the lowest, Dresden the highest figure.

The average annual suicide rate in countries of the world per 100,000 persons living is given by Barker as follows: Saxony, 31.1; Denmark, 25.8; Schleswig-Holstein, 24.0; Austria, 21.2; Switzerland, 20.2; France, 15.7; German Empire, 14.3; Hanover, 14.0; Queensland, 13.5; Prussia, 13.3; Victoria, 11.5; New South Wales, 9.3; Bavaria, 9.1; New Zealand, 9.0; South Australia, 8.9; Sweden, 8.1; Norway, 7.5; Belgium, 6.9; England and Wales, 6.9; Tasmania, 5.3; Hungary, 5.2; Scotland, 4.0; Italy, 3.7; Netherlands, 3.6; United States, 3.5; Russia, 2.9; Ireland, 1.7; Spain, 1.4.

The causes of suicide in European countries are reported as follows: Of 100 suicides: Madness, delirium, 18 per cent; alcoholism, 11; vice, crime, 19; different diseases, 2; moral sufferings, 6; family matters, 4; poverty, want, 4; loss of intellect, 14; consequence of crimes, 3; unknown reasons, 19.

*Homicides.* — Italy takes the lead of European nations, with an average annual crop of murders of 2470, a ratio per 10,000 deaths of 29.4; Spain follows, with a ratio of 23.8, and 1200 murders; Austria, ratio of 8.8, and 600 murders; France, ratio of 8.0, and 662 murders; England, ratio of 7.1, and 377 murders. The figures, however, represent actual murders, not homicides from all causes, as do those in the United States table.

*Illegitimacy.* — Of each 1000 births, the number illegitimate, according to statistics published in London, 1892, were: Russia, 27; Ireland, 28; Holland, 33; England and Wales, 46; Switzerland, 47; Italy, 73; Norway, 74; Scotland, 79; Prussia, 80; France, 84; Hungary, 85; Belgium, 88; Denmark, 93; Sweden, 101; Saxony, 125; Bavaria, 141; Austria, 147. No accurate statistics for the United States exist. The lowest rate in Europe is that of Connaught, in Western Ireland, 7 per 1000. — *Dr. Albert Leffingwell, Summit, N.J.*

3. When phenomena indicate a marked departure from the ratio of frequency as determined by prior observation, or by theoretical considerations, then it is ordinarily inferred that a new cause has become operative, not before existent, or, if present,

its effect neutralized. For instance, we would naturally expect a die to show the face three, on an average, about once in six throws. But if it repeatedly turns up three in succession, and no other number appears, or appears but rarely, we are warranted in inferring that the die is loaded. The number of homicides in the United States in 1894 far exceeded the annual number observed for the several years preceding. This discrepancy is easily accounted for by the fact that the natural number was swollen by the deaths caused by the strikers and rioters in the month of July of that year. So also a marked departure from the annual death-rate of such a city as New York is at once an urgent suggestion to the Board of Health to start investigations that will unearth the hidden cause that one is constrained to believe must be present. Such causes as defective drains, prevalence of epidemics, etc., are again and again found to accompany an increase of the average death-rate.

Under such circumstances, the method of investigation, when practicable, which should be pursued, is to endeavor to break up the total into smaller groups of a specific nature. Thus, if the death-rate for the year is appreciably increased, examine the death-rate per month. See if any month shows a marked departure from the average. If so, this will suggest a careful investigation of the circumstances and characteristics of the month in question. Or it may be possible to make a geographical distribution of the total over different sections of the city under investigation. Some special locality may

indicate an unusually large death-rate. Investigation, therefore, at that point may reveal a lurking cause of disease, otherwise unnoticed.

By similar considerations also, it is often possible to distinguish between a chance coincidence, and a determinate cause which has produced the event in question. For, if the possibility of some one definite cause is considered out of the question, and the origin of the event is found among complex phenomena of such a number and variety that they may form an indefinite number of combinations, only one of which can possibly produce the event in question, then the probability that the event has actually been produced by such a chance combination is extremely small. We are then thrown back upon the other hypothesis, that, instead of one out of many possible combinations, there is some one determinate cause operative in the case. Its nature may not be definitely indicated, but at least the possibility of its presence is suggested.

This line of reasoning is illustrated in the following account of the discovery of the existence of iron in the sun, in the researches of Bunsen and Kirchhoff: "On comparing the spectra of sunlight and of the light proceeding from the incandescent vapor of iron, it became apparent that at least sixty bright lines in the spectrum of iron coincided with dark lines in the sun's spectrum. Such coincidences could never be observed with certainty, because, even if the lines only closely approached, the instrumental imperfections of the spectroscope would make them apparently coincident, and if one

line came within half a millimetre of another, on the map of the spectra, they could not be pronounced distinct. Now the average distance of the solar lines on Kirchhoff's map is two millimetres, and if we throw down a line, as it were by pure chance, on such a map, the probability is about $\frac{1}{2}$ that the new line will fall within one-half millimetre on one side or the other of some one of the solar lines. To put it in another way, we may suppose that each solar line, either on account of its real breadth, or the defects of the instrument, possesses a breadth of one-half millimetre, and that each line in the iron spectrum has a like breadth. The probability, then, is just $\frac{1}{2}$ that the centre of each iron line will come by chance within one millimetre of the centre of a solar line, so as to appear to coincide with it. The probability of casual coincidence of each iron line with a solar line is in like manner $\frac{1}{2}$. Coincidence in the case of each of the sixty iron lines is a very unlikely event if it arises casually, for it would have a probability of only $(\frac{1}{2})^{60}$ or less than one in a trillion. The odds, in short, are more than a million million millions to unity against such a casual coincidence. But on the other hypothesis, that iron exists in the sun, it is highly probable that such coincidences would be observed; it is immensely more probable that sixty coincidences would be observed if iron existed in the sun, than that they should arise from chance. Hence, by our principle, it is immensely probable that iron does exist in the sun."[1]

[1] Jevons, *Principles of Science*, pp. 244, 245.

This principle is also illustrated in instances of circumstantial evidence. In such cases, the observed combination of so many diverse circumstances, even as regards an indefinite number of minor details, precludes the hypothesis of casual coincidence, and suggests some one definite cause that will prove a unifying principle of explanation of all the attendant circumstances. As Mr. Justice Bullen says: "A presumption which necessarily arises from circumstances is very often more convincing and more satisfactory than any other kind of evidence. It is not within the reach and compass of human abilities to invent a train of circumstances which shall be so connected together as to amount to a proof of guilt without affording opportunities to contradict a great part, if not all, of these circumstances."

The following account, taken from *The New York Law Journal*, illustrates the probative force of circumstantial evidence: —

In Nicholas *v.* Commonwealth (March 1895, 21 S. E. R. 364) the Supreme Court of Appeals of Virginia sustained a conviction of murder, the criminal agency being established by circumstantial evidence. The following extract from the opinion presents the main facts which implicated the defendant: —

"On the 8th day of December, 1892, Philip Norman Nicholas, the plaintiff in error, one James Mills, and his wife, Anna A. Mills, and their three small children, were living in the upper part of Henrico County, on a farm known as the 'Wickham Place,' about one mile from James River. Nicholas was the renter of this farm, and cultivated it on shares. He was himself, however, chiefly engaged as

a trapper, having a number of traps set along both sides of the river. He employed James Mills, with whom he lived, and one William Judson Wilkerson, as subtenants, to do the farm work, for a portion of his share of the crops. Wilkerson lived with an aged mother in a small house very near to Mills' house — near enough to see into the windows of one house from the other. Philip N. Nicholas, the prisoner, was an unmarried man, and lived in a room of the house occupied by James Mills and his family. The evidence shows that on the night before the drowning, the prisoner, James Mills, and William J. Wilkerson were together at the house of Mrs. Wilkerson, the latter's mother, and there arranged and determined upon a trip across the river the next morning, to take a bee tree. This expedition was suggested, planned, and carried out by the prisoner. Wilkerson was very unwilling to go, and finally consented at the suggestion of his mother, who said that, as Mr. Nicholas seemed so anxious for him to go, he had better do so. Mills was unwilling to go unless Wilkerson went. Wilkerson said he would rather plough than go. The prisoner replied, 'If you will go, you shall not lose anything.' In the course of conversation which resulted in this expedition being agreed upon, both Mills and Wilkerson stated, in the presence of Nicholas, that they could not swim, and were very much afraid of water; that they did not like water more than knee-deep. The fact that they could not swim was generally known to their friends. It is further shown that it was the habit of Nicholas to go every morning, early, to the river, to examine his traps. And it appears from the evidence that on the morning of the day the drowning occurred he went to the river about daylight, and returned about breakfast time, and, when questioned about it, said: 'I did not go to my traps this morning. I was sick.' He afterwards told Mrs. Wilkerson he did not catch anything. Everything being in readiness to carry out the plan for the day, these three men started from home about nine o'clock in the morning, equipped with everything necessary for

taking the bee tree ; having with them two buckets holding two and one-half to three gallons each for the honey, two axes, one hatchet, and a piece of netting to protect the person from the bees. The boat used belonged to one Joseph Bruin, and on their way to the river an uncle of the owner was asked if they might use the boat, and was told they could get the key which unlocked the boat from its fastening to the bank from Bruin, the owner. The prisoner replied that he had a key of his own, and had often used it before without permission. It appears that they landed on the Chesterfield side of the river, at a point one mile and a half from where any one lived, and proceeded to the bee tree, which was one mile from the point of landing. Investigation showed that there were no tracks about the point of landing but those of the three men going from and returning to the boat. It further appears from the statement of the prisoner that after reaching the tree they concluded not to cut it, because it was a large tree, near the main road, and might get them into trouble, and for the further reason that the hole was small, and it might not have any honey in it anyhow. The tree was afterwards cut by order of the Magistrate and found to be full of honey. It further appears that the boat was a small one, about ten feet long and about two and one-half feet wide, and that both in going over and returning the prisoner sat in the extreme rear of the boat, with his face to the front, and that Wilkerson and Mills sat in front of him, with their faces to the front and their backs to the accused. This position of the parties the prisoner admitted very reluctantly, when questioned about it. When returning, and about fifty yards from the Henrico shore, the boat suddenly filled with water, and Mills and Wilkerson were drowned, and the prisoner swam to shore. The next day the Magistrate of the district was notified of the occurrence, and an investigation was set on foot. The boat was gotten out of the water, and it was found that immediately under the seat where Nicholas sat there were three holes, freshly bored with an inch and a half auger. The

evidence of the owner of the boat shows that on Tuesday evening, the 6th of December, he used his boat, and it was sound. It was taken by Nicholas for this fatal trip Thursday morning, the 8th of December. Further investigation discovered fresh pine shavings corresponding to size of the holes and to the wood the boat was made of, which had been thrown into the water, but had drifted upon the shore near the point where the boat had stood fastened to the Henrico side. There were also found corn-cobs which had been cut to exactly fit the holes in the boat, which had also drifted to the same point. It was shown that the prisoner had in his possession an auger just the size of the holes. This the prisoner at first denied, but afterwards said that it must be about the place somewhere. Diligent search was made for the auger, but it was never found.

"Taken together, the case is an interesting illustration of the conclusive probative force of circumstantial evidence, provided there is enough of it. The old saying that 'murder will out' is almost unexceptionally true as to murders of elaborate stealth and complexity of detail. Once let a clue be obtained to the chain of causation and motive, and the mystery unravels almost of itself. It is quite natural that most of the elaborately planned murders of recent times should have been by poison. And the Harris, Buchanan and Meyer cases in New York disclose how comparatively easy detection and conviction are in crimes of such class. It is significant that two of the greatest enigmas in American criminal annals during the last quarter of a century have been the Nathan murder and the Borden murder. In both cases the killing was done not by methods calculated to conceal the agency of a murderer, but in the most primitive and brutal manner. No traceable physical clue to any particular person was left, and we are inclined to believe that in both cases the connection of the murderer with the crime was merely casual or accidental."[1]

[1] *The New York Law Journal*, Thursday, May 2, 1895.

In the various illustrations which have been given we find that the theory of probability provides a method of dealing with phenomena which cannot be subjected to the ordinary inductive methods. The phenomena are so complex that a specific cause cannot be determined, for the real cause in question is a correlation of many diverse forces, and if only a few instances are examined no causal connection will be disclosed; it is necessary, therefore, to deal with large numbers, statistical averages, etc., in order to detect an emerging relation of a causal character, expressed by a constant ratio. This ratio once determined, it becomes a further test, as we have already seen, when the results widely depart from it, to suggest the presence of a new force outside of the combinations to which the effect would be naturally referred according to the indications of the probability-ratio. This latter mode of inference is akin to the method of residues, for the inference in question is based upon the fact that the probability-ratio will account for only a certain frequency of occurrence of the event under investigation; a marked excess must be accounted for by positing a definitely operative cause. And if an antecedent of such a nature is known to be present, the suggestion at once rises in our thought that this in all probability is the cause producing this excess in the results.

## CHAPTER XVI

### Empirical Laws

There is a class of laws which are intermediate between a universal, inductively grounded by scientific determination, and a law of tendency which is the expression of the probability of the happening of an event in spite of recognized exceptions. These are laws which have been observed to obtain under given conditions of time, place, and circumstance, and yet the causal relation not sufficiently determined to warrant a necessary extension of the same to a sphere beyond that wherein it has been observed to be operative. Such laws are known as Empirical Laws. We have, therefore, three classes of laws of varying degrees of probability. The first is where there has been a scientifically determined causal connection between antecedent and consequent; and not only have no exceptions been noted, but the possibility of there being an exception has been eliminated by strict experimental methods. The second is where the regularity of sequence has been broken by actual exceptions, and the result of the observations of instances gives an indication only of the relative frequency of occurrence and failure which will probably characterize

other events of that nature. The third class and, as has been said, an intermediate class, comprises all expressions of uniform sequence or coexistence, where no exception whatsoever has been noted, and yet there is no ground for necessitating a universal expression of the observed uniformity. There is here always a possibility of an exception appearing, or of an exception having been overlooked. This produces an element of uncertainty which pervades all phenomena of this sort.

There are several kinds of empirical laws, as follows: —

1. Where the causal relation is in process of scientific determination; a uniform connection between phenomena has been observed, and as yet has not been proved. All laws, finally determined as expressions of causal connection, pass through this empirical stage. Some expressions of uniform relations never pass beyond this stage, because, as we have seen, the nature of the phenomena may be such as to preclude all experiment or even indirect verification.

Empirical laws may become ultimate laws or derivative laws, as the case may be. Ultimate laws are those wherein the causal relation between a simple antecedent and its corresponding consequent has been scientifically determined in terms of their exact quantitative variation, and expressed in the simplest form possible. The derivative laws, however, as the name indicates, are more concrete expressions of the ultimate and simpler laws to which they are referred as special cases.

An empirical law may be proved directly an ultimate law, or be proved a derivative law directly traceable to an ultimate law, as its basis, or logical ground. We may observe that a glass of ice-water always shows drops of moisture on its outer surface. This uniformity as thus expressed has the force only of an empirical law. No attempt having been made, as yet, to explain the presence of the moisture, its empirical nature is evident. But as soon as the moisture on the glass is traced to the condensation of the moisture in the atmosphere owing to the difference of temperature between the atmosphere and the cold surface of the glass, we have the empirical law becoming a derivative law; that is, the expression of a uniform sequence directly traceable to the more ultimate law of the saturation and condensation of vapors. The progress of scientific and logically accurate thought is always marked, therefore, by the resolution of empirical generalities into derivative and ultimate laws.

2. The character of an empirical law is attached to the relation existing between antecedent and consequent, when that relation is a complex one in which a simple causal relation is so involved with other elements entering into combination with it, that its real nature is thus hidden and cannot readily be disclosed. This class includes all causal relations due to collocations of various kinds that are necessary to produce the required effect. As Mill has pointed out: "It is the nature of an empirical law that we do not know whether it results from

the different effects of one cause or the effects of different causes. We cannot tell whether it depends wholly upon laws, or partly upon laws and partly upon a collocation. If it depends upon a collocation, it will be true in all the cases in which that particular collocation exists. But since we are entirely ignorant, in case of its depending upon a collocation, what the collocation is, we are not safe in extending the law beyond the limits of time and place in which we have actual experience of its truth. Knowing of no rule or principle to which the collocations themselves conform, we cannot conclude that because a collocation is proved to exist within certain limits of place or time, it will exist beyond those limits."[1]

There are many illustrations of such observed generalities where the effect is due largely, if not altogether, to collocations. The effect of certain medicines upon the human system, the opening and shutting of some flowers at certain hours of the day, the local action of tides at various places on the earth's surface, the adaptation of certain plants to a peculiar kind of soil, the reappearance of some chronic diseases, as hay-fever, at the same season each year, even to the very day of the month, all such generalities have merely an empirical weight, and the effects mentioned are largely due to collocations that cannot be definitely determined. So also certain laws or customs may have proved beneficial in the countries in which they have been tried, and yet, in countries where condition and

[1] Mill, *Logic*, Book III. Chapter XVI. § 4.

circumstance are radically different, they may fail wholly of beneficial results. There may be also certain industrial circumstances which in one country might be conducive to prosperity, and in another country to adversity. Certain agricultural methods which in one section of the country tend to an increase of productive power, in another might prove a complete failure. A governmental policy may in one country lead to unparalleled success; in another, however, a like policy might lead to disastrous results.

The famous formula of Malthus, that population tends to increase in a geometrical progression, whilst the means of subsistence can only increase in an arithmetical progression, can have only an empirical force. Its extension into an indefinite future is unwarrantable. As is known, production has increased enormously and at a ratio vastly greater than any contemplated by Malthus as at all in the range of possibility. Many causes, on the other hand, may combine to check the rapid increase of population. The collocations here are so complex as to defy any definite prediction. This is true of all tendencies which are due to present social conditions; the conditions themselves may so vary in time to come as to change totally the accepted generalizations of to-day. Their empirical character is, therefore, most evident.

3. A third class of empirical laws comprises all those generalizations which represent an aggregate of properties in the same individual. In all such cases no causal relation has been specifically de-

termined between the properties themselves, or between the properties and the whole in which they coinhere. Outside of our experience, the properties observed might be materially changed, and yet not affect the integrity of the concept in general. A proposition such as all swans are white can have only empirical force; for beyond our experience, the discovery of black swans would forbid the proposition being regarded in the light of a universal. Many properties of substances are thus referred to the nature of the substance itself as their ground, and yet because the exact causal relation is not determined the connection can be considered only as an empirical one. In other words, reference to some ground as explanation of a phenomenon, without explaining why or how such reference is made, has always the force of an empirical law only. The folowing are empirical generalizations of this nature. Copper is ductile; steel is elastic; glass is brittle and transparent; the compound silicates of alkalies and alkaline metals are transparent; and other instances of like nature that can be multiplied indefinitely.

In the sphere of biology, Mr. Spencer has drawn attention to the fact that "during the era in which uniformity of many quite simple inorganic relations was still unrecognized, certain organic relations, intrinsically very complex and special, were generalized. The constant coexistence of feathers and a beak, of four legs with an internal bony framework, are facts which were, and are, familiar to every savage. Did a savage find a bird with teeth, or a mammal clothed with feathers, he would

be as much surprised as an instructed naturalist. Now these uniformities of organic structure, thus early perceived, are of exactly the same kind as those more numerous ones later established by biology. The constant coexistence of mammary glands with two occipital condyles to the skull, of vertebræ with teeth lodged in sockets, of frontal horns with the habit of rumination, are generalizations as purely empirical as those known to the original hunter. The botanist cannot in the least understand the complex relation between papilionaceous flowers and seeds borne in flattened pods; he knows these and like connections simply in the same way that the barbarian knows the connections between particular leaves and particular kinds of wood."[1] Such knowledge as Mr. Spencer here describes is a knowledge of the coexistence of two phenomena in their totality which resist all attempts to analyze into their component parts. Moreover, laws which are but general descriptions of correlated events have the same force as the descriptions of coinhering attributes of substances. They, too, rank as empirical generalizations. The successive stages in the growth of a plant from seed to flower and fruit, the embryonic as well as the post-natal developments in animal life, the habits and instincts of animals, — all these are descriptive generalizations without any attempt at causal determination.

4. All generalizations expressed in terms of probability only, because of known exceptions, rank as

[1] Spencer, *Classification of the Sciences*, p. 53.

empirical laws. Here, even in the time, place, and circumstance of observation, the law has not been found always valid. The significance of an empirical law, if we allow this latter class to be included under them, is evidently that of the contradictory of a law which is the result of a causal determination. Every generalization not causally determined is then to be regarded as an empirical law. There is, however, a narrower usage of the term which does not include this latter class; namely, a restriction of the term empirical law to signify the expression of a relation which has been found constant throughout the sphere of observation, and yet where there exists no known causal ground by reason of which we would be warranted in inferring the continuation of this relation in a sphere beyond that already observed. We might add that with this there is also the expectation, greater or less, according to the circumstances attending the phenomena, that the generalized experience will be further confirmed by subsequent observation in a wider sphere. This restricted meaning of an empirical law is the one generally understood, unless it is implied to the contrary.

An empirical uniformity generally results from the method of agreement. Observed instances, even so selected as to vary the antecedents as much as possible, cannot alone establish a law of uniformity that shall have universal validity. The method of agreement, we have seen, needed to be supplemented by the method of difference if possible, or by an hypothesis capable of subse-

quent verification. An empirical law is, therefore, due either to some deficiency in method, or to the natural limitations of our knowledge.

The element of uncertainty attached to all inferences depending upon the extension of an empirical law into unknown territory, it has been insisted upon in several quarters, may apply equally as well to all inferences depending upon the results of the inductive methods even when most scientifically determined. It is contended that even a causal relation, however firmly grounded, and however simple may be its nature, nevertheless presents an empirical character. It may give assurances of a high degree of probability, but can never produce absolute certitude in our minds. Mr. Venn, for instance, has styled his work on induction *Empirical Logic*, that by the title he might indicate his point of view in this regard. He says in the preface to his work: "By the introduction of the term empirical into the title, I wish to emphasize my belief that no ultimate objective certainty, such as Mill, for instance, seemed to attribute to the results of induction, is attainable by any exercise of the human reason." Regarded in this light, all laws are empirical.

The distinction, however, between empirical laws in the sense generally understood, and laws expressing causal relations scientifically determined, is a real distinction, and a significant one as well. And this must not be overlooked; and it cannot be obliterated by any shifting of the point of view. For, to doubt the validity of an empirical

law when extended to a sphere beyond that which has been observed, casts a reflection merely upon one's ability adequately to determine the connections existing between the various elements involved in the particular phenomena under investigation. This is, however, no confession of inability to discover the causal connections of phenomena in general, in such a manner as to determine laws of universal validity. To say that all laws have only empirical significance is to reflect upon the basal postulates of knowledge. Our world is the world as we know it, the world of our consciousness. To discredit the uniformities and regularities therein existing, and which find expression in universal laws, is to discredit that which we feel obliged to think in order that our world of knowledge, regarded as a system, may remain consistent with itself, that is, part to part, and part to whole. We must, therefore, regard an empirical law not as the final form of knowledge, or the goal of inductive research, but rather as marking a transition stage towards complete causal determination. And even when, owing to the nature of certain phenomena, we are not able to pass beyond this transition stage of empirical determination, nevertheless, such instances by contrast bear unimpeachable testimony to the fact that there are other phenomena of such a nature that it is possible to subject them to an analysis which will disclose causal connections of such a character as to form a basis for the formulation of universal laws.

## CHAPTER XVII

### Fallacies

A consideration of the various kinds of inductive fallacies, and their characteristic features, may be regarded as the obverse representation of the general theory of induction. From the one point of view we consider the positive conditions of true inductive inference; from the obverse point of view we regard the various breaches of these inductive conditions. The discussion of fallacies, therefore, indicates no progress in the elucidation of the subject under consideration; it rather serves to emphasize distinctions and requirements already indicated by presenting them in a new light and from a different angle. The subject of fallacies is generally treated by exhibiting through various illustrations the cases in which the positive conditions of inductive inference have failed of satisfactory fulfilment. Such illustrations of the infringement of the requirements of valid induction, I have endeavored to incorporate in the body of the text in connection with the exposition of the various conditions and requirements themselves. In this chapter I shall attempt to indicate those fallacies especially which are due to the psychological disturbance of our

normal logical processes. An enumeration of these tendencies, partly psychological and partly logical, may serve to impress upon us the danger of falling into easy errors, to which the human mind generally is liable. These errors emerge in the various mental processes. They are as follows: —

I. Errors of Perception.
II. Errors of Judgment.
III. Errors of the Imagination.
IV. Errors of the Conceptual Processes.

I. *Errors of Perception.* — Observation is the instrument of research pre-eminently, in all inductive inquiry. Experiment is but a method for increased facility and accuracy of observation. We may say, therefore, that all the data of inductive inference are furnished by this one process, observation. Any derangement of our powers of observation will affect the nature of the data, and therefore the nature of the results of induction. It becomes, therefore, all important that we should be appraised at least of the various circumstances whose tendency is to operate in the midst of the perceptive processes as disturbing forces. We have the following possibilities of error in the sphere of perception: —

1. Errors due to a failure to take in the whole field of vision. There may be portions omitted which possess a determining significance as regards the object of investigation. Thus exceptions may be overlooked that might have an important bearing upon some received hypothesis; or a fact might be passed by which, if known, would prove highly sug-

gestive. Various devices have been employed to enlarge the sphere of observation beyond the natural limits of the senses. As, for instance, sounds which are inaudible to us may be detected by means of a sensitive flame; the telescope, the microscope, serve to render the distant near, and the small large. It had been noted that there was a sudden elongation of an iron wire at a particular temperature whilst under longitudinal strain during the act of cooling from a red heat; an additional circumstance was noted by Professor Barrett when performing the experiment in a darkened room, namely, that at the moment of elongation the wire suddenly evolved heat, and exhibited a visible and conspicuous momentary glow of redness.[1] This circumstance it would be impossible to note unless in a darkened room. Thus, a prominent characteristic of scientific observation is the endeavor to extend continually the sphere of observation. Here also much depends upon the mental habit. There are some who naturally see wider and farther than others. And it is absolutely necessary that the true observer should cultivate with all assiduity such a habit when it is not a natural possession. There is a slovenliness in observation which gives to the inferences based upon its results a color of indefiniteness and inaccuracy, and which proves a fertile source of error.

It also often happens, that, owing to the mind being prepossessed by a certain idea or theory, research will be thereby restricted to a limited region,

[1] Gore, *The Art of Scientific Discovery*, p. 321.

and neighboring regions be wholly overlooked. An open-eyed vision, in spite of all preconceptions or prejudices, is the prime requisite for securing from all quarters the greatest possible array of facts that may in any way tend to the formation of a clearer and more adequate judgment.

2. A second error of observation arises from an opposite mental habit, a failure properly to concentrate the attention upon the relevant facts and so to discriminate as to exclude from consciousness, for the time being at least, all irrelevant details. The lack of such a discriminating faculty leads either to error, or to the dearth of all significant results. It is necessary to avoid either extreme, so that there may be a sweeping survey of all the possible facts relevant to the subject under investigation, combined at the same time with a concentration of attention that is the prerequisite of a deep insight into the inner connections and interrelations of these facts. There must be a depth as well as a wideness of vision.

There are also errors arising from a failure to note significant differences in phenomena that present striking surface resemblances. Here the closest scrutiny is necessary. The older chemists could not distinguish potash from soda; baryta and strontia were formerly confounded together, so also potash and cæsia. Throughout the whole realm of scientific research, it should be ever kept prominently in mind that surface differences may hide essential resemblances, and that surface resemblances may hide essential differences.

3. Errors may arise from apperceptive projection. Here the objective elements of perception combine with the subjective, so that the complete perception may contain elements which do not correspond with reality. The mind thus projects upon the field of vision its own coloring. We see often that which we wish to see, and fail to see that which we do not wish to see. When palladium was originally made known to the public, Chenevix proceeded to examine it, prepossessed with the idea that it was an alloy of some two known metals. This idea was so projected upon his experiments, that he at last came to the conclusion that it was a compound of platinum and mercury. Chenevix was led into an error of observation, as was afterwards proved by Dr. Wollaston, who himself had obtained palladium from the solution of crude platina in aqua regia.[1] This error of observation was due to the fact that he approached the experiments with a fixed idea in his mind as to what they should prove; and being determined to see evidences of this in the phenomena, he unconsciously read into them that which was not really there.

II. *Errors of Judgment.* — These errors occur in the interpretation of the data of perception. For that which is observed must be referred to its appropriate place in the body of knowledge regarded as a system, in which, part must fit to part, and part to whole. Inaccurate reference results in manifest imperfections and incongruities in that part of the system of knowledge to which the ref-

[1] Gore, *The Art of Scientific Discovery.*

erence has been made. And the inferences based thereupon are naturally affected by this fundamental error of judgment. These errors are as follows: —

1. Errors due to false associations. Here, where artificial or superficial associations are interpreted as though they were real causal connections, the mistake may prove most serious. The most fertile source of such fallacies is the wrong interpretation of space and time associations, regarding mere contiguity in space and time as evidence of causal connection. Under this head may be classed the fallacies, *non causa pro causa*, and *post hoc ergo propter hoc.* Prosperity, for instance, following the enactment of certain industrial or tariff measures, is often attributed as the effect of the same, merely because they appear in striking sequence. However, it may be that the prosperity has followed in spite of the laws and not on account of them.

2. Errors of judgment due to emotional perturbations. When the intellect is deflected from its true pointing by passion, or prejudice, or superstition, or any strong emotion, the consequent judgment is the resultant of two forces, rather than the expression of one. As Bacon says: "The human understanding resembles not a dry light, but admits a tincture of the will, and passions which generate their own systems accordingly; for man always believes more readily that which he prefers; his feelings imbue and corrupt his understanding in innumerable and sometimes imperceptible ways."[1]

[1] Bacon, *Novum Organon*, Book I. Aphorism XLIX.

The necessity of judging in a "dry light," as far as possible, is especially emphasized in the ethical positions of Adam Smith, and later of Mr. Sidgwick. Adam Smith contends that one's duty must be estimated from the standpoint of an impartial spectator and critic. That is, man must, as it were, step out of himself, leaving feeling behind, and judge of himself and of his duty from a purely objective point of view. So also Mr. Sidgwick says that one of the chief difficulties in the utilitarian position, namely, the discrepancy between the egoistic and altruistic claims upon our activities, cannot be harmonized satisfactorily, when stated as a problem of mere feeling. Here again man must eliminate feeling and judge of himself merely as one among many, where each counts for one and no one for more than one. In the light of pure reason he may be able to see that the good of all is his highest good. But when that dry light is colored by feeling, such judgment is impossible.

Faraday, in his *Observations on Mental Education*, has borne testimony directly to the necessity of eliminating feeling from our judgments. He says: "The tendency to deceive ourselves regarding all we wish for should be kept in mind, and the necessity also of resistance to these desires. The force of the temptation which urges us to seek for such evidence and appearances as are in favor of our desires, and to disregard those which oppose them, is wonderfully great. In this respect we are all more or less active promoters of error. I will

simply express my strong belief that that point of self-education which consists in teaching the mind to resist its desires and inclinations until they are proved to be right, is the most important of all, not only in things of natural philosophy, but in every department of daily life."[1]

3. Errors of judgment due to the common frailties of human nature. Such errors Bacon has styled "Idols." His enumeration is not only complete, but is classic in its way, and therefore I quote it at this place: "Four species of idols beset the human mind, to which, for distinction's sake, we have assigned names, calling the first Idols of the Tribe, the second Idols of the Den, the third Idols of the Market, the fourth Idols of the Theatre.

"The formation of notions and axioms on the foundation of true induction is the only fitting remedy by which we can ward off and expel these idols. It is, however, of great service to point them out; for the doctrine of idols bears the same relation to the interpretation of nature as that of the confutation of sophisms does to common logic. The idols of the tribe are inherent in human nature, and the very tribe or race of man; for man's sense is falsely asserted to be the standard of things; on the contrary, all the perceptions, both of the senses and the mind, bear reference to man and not to the universe, and the human mind is like those uneven mirrors which impart their own properties to different objects from which rays are emitted, and distort and disfigure them.

[1] Gladstone, *Michael Faraday*, p. 128.

"The idols of the den are those of each individual; for everybody (in addition to the errors common to the race of man) has his own individual den or cavern which intercepts and corrupts the light of nature, either from his own peculiar and singular disposition, or from his education and intercourse with others, or from his reading, and the authority acquired by those whom he reverences and admires, or from the different impressions produced on the mind as it happens to be preoccupied and predisposed, or equable and tranquil, and the like; so that the spirit of man (according to its several dispositions) is variable, confused, and as it were actuated by chance; and Heraclitus said well that men search for knowledge in lesser worlds, and not in the greater or common world.

"There are also idols formed by the reciprocal intercourse and society of man with man, which we call idols of the market, from the commerce and association of men with each other; for men converse by means of language, but words are formed at the will of the generality, and there arises from a bad and unapt formation of words a wonderful obstruction to the mind. Nor can the definitions and explanations with which learned men are wont to guard and protect themselves in some instances, afford a complete remedy, — words still manifestly force the understanding, throw everything into confusion, and lead mankind into vain and innumerable controversies and fallacies.

"Lastly, there are idols which have crept into men's minds from the various dogmas of peculiar

systems of philosophy, and also from the perverted rules of demonstration, and these we denominate idols of the theatre; for we regard all the systems of philosophy hitherto received or imagined, as so many plays brought out and performed, creating fictions and theatrical worlds. Nor do we allude merely to general systems, but also to many elements and axioms of sciences which have become inveterate by tradition, implicit credence, and neglect."[1]

All such tendencies, as thus presented by Bacon, clog and hamper the normal functioning of the judgment. The mind must be alert and on guard to eliminate such fatal seeds of error.

III. *Errors due to the Imagination.*—Here the imagination supplies inner connections and relations, lying beyond the sphere of observation, in order to explain the nature of the observed phenomena themselves. The danger here is that the elements supplied in order to make the self-consistent whole do not correspond to reality. The system, regarded as a mental construction, may be complete in all of its co-ordinated parts, and nevertheless possess no objective reality. Under this head fall all loosely constructed hypotheses. In the framing of an hypothesis in general, the imagination functions very largely. It is the inner vision that represents to the mind the things not seen. Moreover, the imagination is peculiarly liable to error, and to swing clear of the trammels of fact, and in the region of pure fancy construct systems

[1] Bacon, *Novum Organon*, Book I. Aphorisms XXXIX. etc.

that rest upon no solid basis of reality. These dangers in detail have been pointed out in the chapter on "Hypothesis."

The most fertile source of error, however, arises from that natural elation of mind upon the discovery even of slight confirming evidence of the truth of the assumed hypothesis. This enthusiasm is apt to magnify unduly an inadequate verification, and to rest satisfied in an hypothesis that is grounded upon an insufficient basis. Thus since the year 1770 more than forty discoveries of new elementary substances have been announced to the world by enthusiastic experimenters, and, in all cases, their discoveries have been proved to be absolutely worthless. For instance, it was confidently announced that Torbern Bergmann, in 1777, had extracted from diamonds what he considered to be a new earth, and called it "terra nobilis." Wedgwood, in 1790, discovered "australia" in sand obtained from the continent of that name; but Hatchett proved it to be merely a mixture of silica, alumina, oxide of iron, and plumbago. In 1805 Richter discovered "niccolanium"; it was afterwards proved to be a mixture of iron, cobalt, nickel, and arsenic. These instances are but a few of the many which characterize the history of chemical research, and stand as conspicuous witnesses of the danger of divorcing fancy from fact.

The imagination, however, properly constrained is most potent in suggesting possible causal relations, in constructing hypotheses, in devising methods of experiment in order to verify them, and in forming

universal concepts in which all the particulars of observation must coinhere. Davy and Faraday were both conspicuous in this mental characteristic. And to this source their eminent discoveries may be traced. Dr. Whewell says, for instance, of Faraday: "In discovering the nature of voltaic action, the essential intellectual requisite was to have a distinct conception of that which Faraday expressed by the remarkable phrase, '*An axis of power having equal and opposite forces.*' And the distinctness of this idea in Faraday's mind shines forth in every part of his writings. He appears to possess the idea of this kind of force with the same eminent distinctness with which Archimedes in the ancient and Stevinus in the modern history of science possessed the idea of pressure, and were thus able to found the idea of mechanics. And when Faraday cannot obtain these distinct modes of conception, he is dissatisfied and conscious of defect."[1]

It is indeed a touch of genius that enables one to grasp and formulate a central idea that will unify and also universalize a large body of seemingly disconnected and incongruous facts. But such an idea must be the expression of the relations actually obtaining, and no subjective fancy projected upon the phenomena themselves, however clever or ingenious such an imaginative creation may be. If one were asked what is the most efficient instrument of scientific research, the answer must be, "The Imagination!" And if one were asked what is the

[1] Whewell, *History of Inductive Sciences*, Vol. III. 3d ed., p. 147.

most fertile source of error, the answer likewise must be, "The Imagination!" It must also be remembered that it is not sufficient merely that an hypothesis should be in harmony with the facts in the case; it must be proved also that the facts are connected with the hypothesis through necessary links.

And it is well also to bear this in mind when arguing against a rival hypothesis that may have been advanced by an opponent who has claimed for it only the possibility of its validity, and who has not affirmed its necessity. It is manifestly unfair, as well as fallacious, to deny the possibility of the hypothesis merely by indicating certain uncertainties connected with establishing it. To contradict possibility, one must prove the hypothesis impossible. Regarding such a conflict between rival hypotheses Ueberweg suggestively comments as follows: "In cases of this kind, it is one of the hardest of scientific and ethical problems to give fair play to one's opponent. Our own prejudices are sure to influence us. Yet the effect of the influence of another's standpoint, when it is reached, is of immense value in scientific knowledge. Polemic easily leads to exasperation; it is easy both to abuse it, and to let it alone because of dislike to the conflicts which it produces; but it is difficult to recognize it, and use it in the right sense as the necessary form which the labor of investigation always takes. Man never attains to a scientific knowledge of the truth without a rightly conducted battle of scientifically justifiable hypotheses, the one

against the other: the scientific guidance of this battle is the *true dialectic method.*"[1]

IV. *Errors of the Conceptual Processes.* — This class of errors arises in the formation of general concepts and their expression in universal laws. The natural tendency of the mind to generalize often leads to ill-considered results. The universal unites many differences into an identity, and the mind will readily minimize the differences in order to form a desired universal; thus disparate attributes may be incorrectly co-ordinated in one and the same system. Herschel has remarked that hasty generalization is the bane of science. And Bacon has said our intellects want not wings, but rather weights of lead to moderate their course.

The method of agreement, when relied upon to the exclusion of further experimental determination, is a fertile source of error in this respect. The possibility of a plurality of causes should be ever kept prominently in mind. One readily assigns an effect to a causal element which is only partially its cause; the consequent generalization is, therefore, incorrect. For instance, it often happens that activities of young animals are described as instinctive and congenital; and universal propositions are founded thereupon. And yet it may be that the activities referred solely to instinct are due partially to imitation. In order to avoid this error and eliminate the factor of imitation, investigators in this line are accustomed to study the activities of animals hatched in incubators and purposely

[1] Ueberweg, *A System of Logic, etc.*, p. 509.

kept from all of their kind. This illustration will serve to show the precautions that must be taken in order to eliminate all possible error from the data which the process of generalization constructs into universal forms. So also inaccuracies in any of the other inductive methods lead to gross errors in the consequent generalizations based upon them.

Under this head, also, are the fallacies resulting from the extension of empirical laws to spheres beyond the experience which they embody and express. This source of error is especially illustrated in laws expressing some quantitative relation between antecedent and consequent; it is a natural supposition in such cases, and yet a very misleading one oftentimes, that a simple proportional relation will exist between phenomena of the same, but with greater or lesser magnitude as the case may be. Bacon gives the following illustrations of this fallacy: "Suppose a leaden ball of a pound weight, let fall from a steeple, reaches the earth in ten seconds, will a ball of two pounds, where the power of natural motion, as they call it, should be double, reach it in five? No, they will fall almost in equal times, and not be accelerated according to quantity. Suppose a drachm of sulphur would liquefy half a pound of steel, will, therefore, an ounce of sulphur liquefy four pounds of steel? It does not follow; for the stubbornness of the matter in the patient is more increased by quantity than the activity of the agent. Besides, too much as well as too little may frustrate the effect,—thus, in smelting and refining of metals it is a common

error to increase the heat of the furnace or the quantity of the flux; but if these exceed a due proportion, they prejudice the operation because by their force and corrosiveness they turn much of the pure metal into fumes, and carry it off, whence there ensues not only a loss in the metal, but the remaining mass becomes more sluggish and intractable. Men should, therefore, remember how Æsop's housewife was deceived, who expected that by doubling her feed her hen should lay two eggs a day; but the hen grew fat and laid none. It is absolutely unsafe to rely upon any natural experiment before proof be made of it, both in a less and a larger quantity."[1]

Another fallacy of the same order often occurs in the inference concerning the interpolated elements of a series whose successive values have not all been observed. The inference extends the nature of the known to the unknown parts, and presumes that the intermediate links between actually observed parts of the series are in accordance with the general nature of the latter. Such inferences very often give correct results, as, in the plotting of a curve, some salient points may be determined according to observed quantitative variations, and the remaining portions supplied, as upon the above supposition. This extension to cover intermediate and unobserved instances may, however, be sometimes very fallacious. For a force may be assumed to be such that its effects increase steadily, and it

[1] Bacon, *Advancement of Learning,* Book V. Chapter II. p. 190.

may be that they operate periodically; interpolation upon one assumed basis when the other is the true one, would of course introduce grave errors. To eliminate such errors, devices have in many cases been resorted to by which a self-registering apparatus will record all successive values of the phenomena under investigation.

Under the fallacies of hasty generalization, naturally fall all provincialisms which arise from a narrow nature and habit of mind. The local traditions and superstitions, the prevailing winds, the social customs and manners, are taken as types of a universal experience. The inferential widening of the circle of a limited experience is always provocative of false inference and misleading results.

We have also false analogies which consist in the extension of our experience of certain phenomena that we have observed to be alike in some respects to include other attributes not observed, concerning which we assume a corresponding similarity; the abuse of final causes may be regarded as a special case of false analogy. Moreover, a tendency to consider causation in the light exclusively of final causes has often retarded the advance of science, in withdrawing the attention and energies of the investigator from a search after physical causes, as, for instance, among the ancients it was declared that the leaves of the trees are to defend the fruit from the sun and wind. Resting satisfied in such an explanation, the precise function of the leaves in the economy of the plant's growth was not further investigated, and thus progress was impossible.

Again, incorrect classification is a source of error. In grouping together disparate phenomena, we have a basis for forming a generic concept that will include incompatible species, or, in other words, a universal that will have evident exceptions. Moreover, if the classification is partial, the resulting laws based upon it will have only empirical force.

I have endeavored in this chapter to indicate errors that are mainly psychological in their origin, for two reasons. In the first place, such errors operate as disturbing forces in the midst of purely logical processes. The data of inference are psychological as regards their source, and errors thus originating affect the inference based upon them, appearing in the final result as logical fallacies. An error of observation becomes an error in the judgment that is based upon the original perception, and perdures in the hypothesis, classification, etc., founded on that judgment, and finally emerges in the conclusions based upon these processes. In the second place, the fallacies that are purely formal, and in the strict sense logical, are not as apt to deceive and mislead the mind. In the material data especially lurk the germs of fallacy. On the theory that it is wiser and also more logical to stamp out an error in its incipiency, I have placed special emphasis upon the various psychological processes as initial sources of error. Moreover, it is more rational to deal with errors of process rather than flaws of product. A machine that turns out imperfect articles could have its imperfections rectified by repairing each article thus pro-

duced; or the machine itself could be readjusted so as to produce the articles without flaw. It is needless to say which method is the more logical, and most satisfactory, as well.

The desideratum is accurate and comprehensive observation; a discriminating judgment formed in the "dry light" of reason; an imagination that has deep insight into the heart of surface appearances; and powers of generalization which transcend observed phenomena by adequately interpreting them.

## CHAPTER XVIII

### The Inductive Methods as Applied to the Various Sciences

The nature of each separate science will determine certain peculiarities of method for that science; and its peculiar method will be largely a matter of growth, as experience accredits or discredits the various results which its operation may attain. It will thus be corrected or supplemented according as it stands the test of achieved results. There are, however, some general features, and especially some natural limitations of the inductive methods, that may be properly indicated.

I. In the first place, the nature of the method used, and its efficiency, as measured by its results, will be found to vary as the nature of the phenomena themselves. Some phenomena admit of analysis and further determination by experiment. Instead of attempting to determine the relation of a complex antecedent to a complex consequent, the antecedent is first separated into its component parts, and one element is tested alone in order to determine its precise effect. The relation can then be expressed between the simple antecedent and simple consequent, as a causal connection; and it

admits, moreover, of a quantitative determination as well. Such a method of procedure by analytical experiment enables us to rise to laws having universal validity. This method is characteristic, especially, of the physical sciences, because the phenomena readily admit of resolution into component parts, and the isolation of one simple force so as to determine its total effect. The physical forces are most readily adaptable to experiment. They therefore afford the widest field for the application of the experimental method of inductive inquiry. Moreover, we may readily predict the results of a combination of simple forces, when we know the laws governing their component elements. The inducto-deductive method, therefore, becomes especially efficient in extending the domain of the physical sciences. Here, also, mathematical analysis and calculation is most valuable as an aid to experimental investigation, and in determining quantitative relations as necessitated by the mathematical laws to which the data gathered inductively must conform.

There are, however, sciences which present phenomena of such a high degree of complexity that an analysis of a complex whole into its separate parts or elements of force is impossible. Moreover, the phenomena cannot be analyzed in this respect either, that a certain part of the complex whole can be indicated as the whole antecedent, and the remaining portion as the entire consequent. The difficulty, therefore, is twofold; it is impossible to separate the complex whole into

two other complex wholes, antecedent and consequent, and still further impossible to separate such antecedent and consequent, even if they could be determined, into their simplest component parts. The phenomena presented are here not in the form of a sequence so often as in that of coexistence, as in the sciences of botany, zoölogy, and the like. Here the methods of analogy and classification must be resorted to, and we obtain descriptive laws as the result.

The forces manifested in the processes of vital growth are especially difficult to determine by experiment; for they not only operate to produce certain effects, but perdure in the effects to produce certain other effects in a process of continuous construction. Separation by mechanical analysis means instant cessation of the process itself. Dissection means death. Here, then, is a natural limitation. Moreover, the laws of development are further modified by external changes. The result of the inner force and the outer influences, working together, complicates the problems to such an extent that the pure inductive methods are well-nigh impossible of application. Resort is then had to determination by statistical methods. Large groups of plants and animals are examined for the purpose of noting tendencies disclosed in the aggregate, but hidden as regards their manifestation in the individual. Here, of course, classification is an aid in disclosing similarities and differences that may suggest hypotheses to explain certain dominant characteristics.

We may, moreover, have merely permanent effects presented in perception, the cause having ceased to act long since. Thus in geology we have facts that have been caused, it is true, but the causes can be discerned only as manifested in the effects, and, therefore, they can be determined only by the method of hypothesis, which may lead to verification or not, as the case may be. Again, certain sciences may suggest problems which concern the explanation or significance, not of particular phenomena within the sphere of that science, but rather the interpretation of the whole body of phenomena which the science in question comprehends. The problem is not solved, therefore, by any attempt in the way of analysis by experiment, but rather in the way of synthesis through hypothesis; that is, the ideal construction of a whole which will unify and account for all facts, or, in other words, to discern the system which underlies and co-ordinates the various particular manifestations. This is seen especially illustrated in the problems which geology and biology present concerning the interpretation of their respective phenomena regarded in the light of their totality. Astronomy also presents a mass of seemingly chaotic phenomena, and yet the aim of this science is to reduce them all to some one self-consistent system.

For instance, Mr. Spencer remarks concerning the geologist: "He does not take for his problem only those irregularities of the earth's crust that are worked by denudation; or only those which igneous

action causes. He does not seek simply to understand how sedimentary strata were formed; or how faults were produced; or how moraines originated; or how the beds of Alpine lakes were scooped out. But taking into account all agencies co-operating in endless and ever-varying combinations, he aims to interpret the entire structure of the earth's crust. If he studies separately the action of rain, rivers, glaciers, icebergs, tides, waves, volcanoes, earthquakes, etc., he does so that he may be better able to comprehend their joint actions as factors in geological phenomena; the object of his science being to generalize these phenomena in all their involved connections as parts of one whole." Mr. Spencer also describes the nature of biology in much the same way. "In like manner biology is the elaboration of a complete theory of life in each and all of its involved manifestations. If different aspects of its phenomena are investigated apart, if one observer busies himself in classing organisms, another in dissecting them, another in ascertaining their chemical compositions, another in studying functions, another in tracing laws of modification, — they are all consciously or unconsciously helping to work out a solution of vital phenomena in their entirety, both as displayed by individual organisms and by organisms at large."[1]

Mr. Spencer makes the distinction between investigation of particular causal relations on the one hand, and, on the other, the interpretation of

[1] Spencer, *Classification of the Sciences*, pp. 19, 20.

the total phenomena of a science as the basis of classification of the sciences. He divides the sciences into those which treat of the forms in which phenomena are known, such as the Abstract Sciences of mathematics and logic (*i.e.* formal logic); and those which treat of the phenomena themselves or the Concrete Sciences. The latter redivide into the Abstract Concrete Sciences, so called, because the specific elements of the concrete phenomena are abstracted from the phenomena, considered as a whole, and so determined as causal laws, relative to particular references within the whole body of phenomena which the science comprehends; these are such sciences as mechanics, physics, chemistry, etc.: the second is that of the Concrete Sciences simply, which regard phenomena in their totalities, as above described. A difference of method is here indicated. The Abstract Concrete Series are to be investigated analytically, that is, by experiment principally, with a view of exhibiting complex phenomena in their simplest terms. The simple Concrete Sciences are to be treated synthetically, that is, by the framing of an hypothesis that will comprehend all particular phenomena in one co-ordinated whole.

The division of Herbert Spencer can only be accepted in a general way as indicating predominant characteristics of the two kinds of sciences. It will not do to lay down hard and fast lines here, for *every* science presents two kinds of problems; the first, to determine particular

causal relations; the second, to co-ordinate all such relations into a self-consistent system which will unify all separate and individual instances. For instance, take the phenomena of light in physics. Particular problems as regards intensity, velocity, composition of light, etc., present themselves; then an underlying problem, How explain all the phenomena of light upon some one single basis regarding the essential nature of light. Hence arose the emission and undulatory theories of light, and all phenomena bearing upon the theory were marshalled in support of one and of the other, until the conflict was conclusively decided. And again, the theory of light, the theory of electricity, the theory of heat, etc., suggest still another problem, How unify all the separate theories in one all-comprehensive theory to which the separate phenomena may be alike referred. Thus every science presents particular problems, and a general problem as well. And herein lies a suggestion that all investigators in any branch of science would do well to bear in mind. Specialization in any one line of particular problems should always lead to a consideration of the relations of these particular problems to the general system of which they are parts. Specialization that does not thus supply its own corrective by the natural insistence of the mind to interpret the particular in the light of more general laws, tends to narrowness of mind and barren results.

II. In reference to method in the sciences it must be observed also that in certain phenomena

the simple theory, which regards the causal connection as a transfer of energy according to the doctrine of the conservation of energy, is further complicated by certain variations and modifications of the energy in the process of transference. When, for instance, a billiard-ball strikes another, and the second ball, by virtue of the impact, receives the energy of the initial moving ball transferred to it, the problem is simplified by the fact that the motion of the first is easily traceable in the second, being a transfer of energy which manifests itself in the same manner in the two cases. However, the problem is complicated at once when in chemistry, for instance, the two combining elements form a third in which the characteristic features of the former are wholly lost in the new form. Here is likewise a transfer of energy, which may have mechanical equivalents, it is true, and yet so radical a change of form accompanies the transfer that it complicates the problems which arise in this science. We have seen how the combined inducto-deductive method often predicts events and the nature of phenomena not yet observed. And yet this becomes most difficult whenever transfer of energy is accompanied by a change in the peculiarities of its manifestation as well. Knowledge of the nature of two elements, and all their separate characteristics, will not be sufficient data for any prediction as to the nature of the compound. Thus chemistry confronts a natural difficulty as regards method, which does not affect physical science generally.

Another difficulty appears in psychology, for here

stimuli from the outer world, expressed in terms of physical energy and quantitatively determined, produce psychical reactions, that cannot be expressed in physical terms. And, on the other hand, processes of ideation produce muscular activities, that may be estimated in physical terms. It has been urged that here the theory of conservation of energy breaks down, that the transferred energy is wholly accounted for by the nerve and brain modifications, and that the psychical accompaniments are wholly unaccounted for upon this basis. They stand out as unexplored remainders.

This objection is met in two ways. One is that the physical and psychical are, as it were, two closed circles, and while simultaneous in their functioning do not mutually interact. This is the theory of the so-called "psychophysical parallelism." It necessitates metaphysical explanations and postulates that seem to complicate rather than simplify the difficulties. The second is the more reasonable, that psychical activity may be radically different from physical and yet the two capable of reacting upon each other, so as to liberate the potential activities in either sphere, and thus initiate a series of causally connected phenomena. Such a theory is buttressed by substantial analogies in the physical sphere itself; namely, that in many phenomena the impinging force is so modified, by the reaction due to the nature of the substance acted upon, as to lose, to all observation at least, its original characteristic features. For instance, friction passes over into electricity because of the nature of

the substance that is rubbed; thunder sours cream, and thus sound vibrations cause effects wholly incongruous to them.

These illustrations might be multiplied through all the realm of physical science. They are so many as to prepare us for realizing the possibility, at least, that physical excitations may produce psychical phenomena in the sense that the outer stimulus calls into activity psychical energies, that thus stirred, manifest themselves according to the forms of their own nature, rather than the forms of the physical phenomena exciting them. Upon such a theory we may proceed, by observation and experiment, to measure duration, intensity, etc., of mental reactions responding to external stimuli. As regards the method here employed, the series is considered as one and complete, so that physical excitations are traced, as it were, through an unbroken causal chain to their psychical effects, and *vice versa.* On the theory of two closed circles, it is difficult to indicate a logical method of experimental inquiry, unless it be further postulated that activities in the one, according to its kind, may induce modifications of the other according to its kind. This reservation is generally insisted upon.

III. It sometimes happens that the phenomena of one science are to be interpreted in the light of the results of another science. Thus the laws of one science become guiding principles in investigating the causal relations existing in another sphere. This can only be done when there is some similarity between the phenomena of the two sci-

ences. This method is especially illustrated in historical explanation. The problem presents a mass of events that must be co-ordinated in a system wherein their several causal relations will be exhibited. And not merely must detached epochs be proved causally interrelated as regards the events occurring in them, but here, also, the special problems give rise to a general problem, to discover in the whole the philosophy of history, and to determine the several historical tendencies in one system whose characteristic features will reveal the fact that "through the ages one increasing purpose runs."

To solve the special and the general problems of history, recourse is had to an analysis of events on the basis of well-established psychological results. The phenomena of history are substantially the activities of man, both in his individual and collective capacities. Events being given, an hypothesis concerning the motives, and ends which actuated them, is framed upon the supposition that men ordinarily are impelled by similar motives under similar circumstances, in order to achieve similar ends. Here the analogies drawn between men of the present and men of the past, or between men moving in the ordinary routine of every-day life and men whose acts may be epoch-making, furnish a basis for historical interpretation. We say that a series of events, perhaps of a very complicated nature, can be explained only by an hypothesis that a well-defined purpose and a strong determined will were fashioning them and moving through them to an end that was in the chief actor's mind from the

beginning. And so the rise of social habits, customs, traditions, laws, the religion, the government, and national institutions of a people have an origin in psychical and not physical elements, a deeper understanding of whose nature and all that it necessitates tends to a clearer elucidation of the problems therein presented. The knowledge of man, the microcosm, is a guiding thread amid the bewildering mazes of the macrocosm. It is possible, of course, for the imagination of the historian to lead him to wander far afield, and invent fanciful motives, purposes, public policies, etc., to explain given events. However, here, as in the physical and other sciences, the hypotheses framed must meet the general requirements and conditions of a valid hypothesis.

IV. There has been a growing tendency in sciences regarded as solely or largely deductive, to correct and supplement the traditional method and results by a more searching inductive inquiry. This is especially true of political economy. The deductive method proceeded to build up a body of doctrine composed of inferences necessitated by a few fundamental premises. The premises were such as the following: The principal motive of action is self-interest; the earth, as man's great supply-house, is limited in extent and productivity; the physical and psychological tendencies of man lead him to multiply his own species with a rapidity which, if not counteracted by obstacles, would bring about an unlimited increase of population. All economic laws were thus deduced from some such fundamental propositions as these. The re-

sults of this deductive method, however, have been brought to the bar of another method for searching examination and judicial sentence. In 1848 Hildebrand, and Knies in 1853, with Roscher in 1854, set forth the principles of the historical school of political economy. They held that an inductive inquiry must be started in order to estimate the physical, ethnical, and historical conditions of a nation and its stage of civilization. These forces, correctly assessed, will give the economic conditions of a particular period of history, or of a particular nation. This is not the place to criticise the tenets of this school, but merely to point out the fact that its influence has been potent in correcting and supplementing the results obtained in a purely deductive manner.

Deduction may give the joint effect of universal psychological impulses, operative under certain natural conditions of environment, etc., provided no disturbing force is present. But the question here is not whether a certain cause, if acting alone, will produce a certain effect; but whether counteracting causes will be present, or modifying causes, as the case may be. To estimate the results of collocations and not simple causes becomes, as we have seen, a complex problem. For its solution recourse must be had largely to statistical methods whereby large aggregates reveal tendencies that are actual and not theoretical merely.

In a similar way, the historical school of jurisprudence, associated with Savigny, has influenced the so-called philosophical school in demonstrating

that results theoretically determined by deduction are constantly modified by the real conditions and limitations of each particular nation's life. The influence of this school is indicated by a significant fact, that when Hegel wrote his theory of law (*Rechtslehre*) he paid more regard to the historical formation of states than did the earlier theorists of natural law.[1]

Again, another illustration of the growing prevalency of inductive method is found in the modern psychological method. The sole method was considered from time immemorial to be that of introspection. Its results, however, were meagre; the method itself was indefinite and lacked certainty and uniformity. Inductive inquiry, therefore, proceeded by its own methods to secure and interpret material in other and various fields. As Professor Ladd says: "The method of psychological science is peculiarly introspective and analytic of the envisaged phenomena called states of consciousness. But it is far broader and more effective than it could be if it were merely introspective. It pushes its analysis of the genesis of the phenomena as far back as possible, by the use of experimental methods, and methods of external observation applied to the whole process of mental evolution (study of infants, of primitive man, and of the lower animals, — evolutionary and comparative psychology). It interprets the psychological life of the individual mind in the light of knowledge gathered concerning the psychical development

[1] Bluntschli, *The Theory of the State*, p. 69.

of the race (the psychological study of literature, society, art, religion, etc.). It lays peculiar emphasis upon abnormal and pathological phenomena of the nervous and mental life (psychiatry, hypnotism, phenomena of insanity and of the criminal classes, etc.). It takes account of the rise and fall of particular forms of psychological theory (the history of psychology). It strives to transcend experience by hypothetical principles of explanation. But in the employment of all these methods this science differs in no important respect from the sciences which deal wholly with physical phenomena. It is only the use of introspection for the possession, and, to some extent at least, for the analysis, of its objects, which makes psychology, as respects its method, different from the other sciences."[1]

In the above, we see that inductive inquiry lays all possible fields of research under tribute to the one end of explaining and correlating psychical phenomena. The systems of ethics also, which are founded upon an *a priori* basis, are becoming more indebted to empirical investigations which have given a richer content to the strictly formal ethic. Advanced psychological research, the study of race characteristics, tribal customs, habits, law, religion, etc., the indications of moral progress, — all give material which, if interpreted by right hypotheses, will throw light upon the theory of ethical principles regarded merely from a speculative point of view. We may conclude, therefore, that the inductive method and the deductive are not mutually exclu-

[1] Ladd, *Introduction to Philosophy*, p. 116.

sive processes. They may be so combined as mutually to strengthen one another. What Bluntschli says of jurisprudence may be applied equally as well to all sciences that claim some *a priori* basis: "The old strife between the philosophical and historical schools in Germany has altogether ceased. Peace was made as early as 1840. Since then it is recognized on all sides that the experiences and phenomena of history must be illumined with the light of ideas, and that speculation is childish if it does not consider the real conditions of the nation's life."[1]

It will be seen how important a factor historical data becomes, in all the sciences that deal with human volition and activities. Whatever hypothesis may be framed, it must correspond to these data, because they represent actual conditions that must be co-ordinated in a self-consistent system, and their nature and relations satisfactorily interpreted.

[1] Bluntschli, *The Theory of the State*, p. 70.

## CHAPTER XIX

### Historical Sketch of Induction

*Socrates* (470–399 B.C.). — We find the beginnings of inductive inquiry in the Socratic or *maieutic* method, that art of mental midwifery by which conceptions were to be delivered from the mass of individual experiences and opinions in which they lie concealed. The Socratic procedure in the formation of conceptions is to question every particular view, and estimate it by bringing together analogous cases, and discovering their natural connections, so as to explicate the general notion which it contains, and thus proceed from comparison of particulars to the framing of general propositions. Socrates' generalizations were many of them hasty, and in his desire to formulate a general conception he overlooked exceptions and minimized difficulties, but in his method there were the germs of truly scientific procedure. The sphere of his method was, however, limited, as he applied it only to the illumination of ethical controversies.

*Plato* (427–347 B.C.). —Plato enriched the Socratic method of induction by removing its limitation to ethical inquiry. Plato was especially concerned with investigating the relations of his "ideas" to each other, and this led to the apprehension of the

logical relations between conceptions, especially as regards their subordination and co-ordination. This forms a basis for classification, — Plato's division of class-concepts or logical genera into their species is a prominent feature of his method. He also suggests the hypothetical method of treating the relations of concepts; namely, to examine a tentatively proposed conception by developing all the possible consequences that would follow from its union with known conceptions. This is in keeping with the inducto-deductive method of Mill and the modern logicians.

*Aristotle* (384–322 B.C.). — Aristotle's name is especially, and it may be said almost exclusively, associated with deductive logic and syllogistic reasoning. Although he did not develop fully the inductive logic, he nevertheless did not ignore it, in some of its essential features at least. He acknowledged the necessity of investigating the starting-point of deduction, namely, the ultimate grounds of proof, and of the principles of explanation. This process he called dialectic. It is a double process that proceeds from the particulars given in perception, and from the ideas current in customary opinion, to discover the general, and then from the general to deduce the particular, which is thereby verified in the process. The former procedure is the reverse of the deductive, and is epagogic or inductive. Induction, according to him, is a syllogism in which the inference that the major belongs to the middle, is mediated through the minor directly; and not indirectly through the middle. Thus, to use Aristotle's illus-

tration, the investigation of the connection between the absence of gall in animals and longevity in a number of instances, as in man, horse, mule, etc., may disclose their coexistence.[1] They are then united directly without mediation of a middle term. If we had given the universal proposition to start with, Whatever animal has no gall is long-lived, and the minor premise that man, horse, mule, etc., are animals having no gall, then the conclusion would follow, therefore they are long-lived. This is the deductive syllogism. The inductive method, on the other hand, starts from particular observation that the horse which has no gall is long-lived, so also the mule, so also man, etc.; therefore, without any middle term, a coexistence is taken as equivalent to a causal relation between these attributes, and the inference is drawn that all animals having no gall are long-lived. Such an inference is valid syllogistically, however, only on the assumption that the instances examined comprise the whole class having the attributes under investigation. This inductive syllogism, therefore, expresses inferences only of complete enumeration.

The form of such a syllogism is as follows: —

Let $S =$ minor term,
$P =$ major term,
$M =$ middle term.

This, that, and the other $S$ is $P$.
This, that, and the other $S$ is all $M$.
$\therefore$ All $M$ is $P$.

[1] Aristotle, *Prior Analytics*, II. xxiii.

Here it will be observed that the particular instances comprising the minor term $S$, when summed up, equal the middle term. There is no inference in this if we have regard to the strict sense in which the word is used. Aristotle, indeed, considered the only scientific induction to be the so-called perfect induction, and says that to generalize many experiences of the same kind is admissible only when there is no contrary case. The thought that causal connection enables us to generalize is stated by Aristotle, but, as Ueberweg says, it "does not attain to a fundamental significance in his logical theory."[1]

*The Precursors of Bacon.* — The revolt against the scholasticism of the Middle Ages and the fetters of the Aristotelian logic was many-voiced, culminating, however, as regards the emphasis placed upon induction as a scientific method, in the works of Francis Bacon.

Foremost among the early champions of inductive inquiry we find Roger Bacon, born in 1214, a Franciscan monk, yet devoted heart and mind to the cause of science. His *Opus Majus* was published first in 1733 by Dr. S. Jebb, principally from a manuscript in the library of Trinity College, Dublin. This work is characterized by a spirit of protest against authority in general, and that of Aristotle and his logic especially. He recommends mathematics and experiment as the two great instruments of scientific investigation. In this particular it is interesting to note his antici-

[1] Ueberweg, *Logic*, p. 479.

pation of the modern mathematico-physical modes of scientific inquiry. The following quotation will give an indication of his spirit and aims: —

"Experimental science, the sole mistress of speculative sciences, has three great prerogatives among other parts of knowledge: First, she tests by experiment the noblest conclusions of all other sciences; next, she discovers, respecting the notions which other sciences deal with, magnificent truths to which these sciences of themselves can by no means attain; her third dignity is, that she by her own power, and without respect of other sciences, investigates the secrets of nature."[1]

*Leonardo da Vinci* (1452–1519). — Leonardo combined in one personality many brilliant talents, being eminent as sculptor, painter, architect, engineer, astronomer, and natural philosopher. His works, unpublished, exist in manuscripts in the library of the Institute at Paris. He expresses himself very clearly and emphatically concerning the relation of experience to speculation: "Theory is the general; experiments are the soldiers. We must consult experience, and vary the circumstances till we have drawn from them general rules; for it is she who furnishes true rules. But of what use, you ask, are these rules? I reply, that they direct us in the researches of nature and the operations of art. They prevent our imposing upon ourselves and others, by promising ourselves results which we cannot obtain. But see the absurdity of men!

[1] Whewell, *Philosophy of the Inductive Sciences*, Vol. II. p. 333.

They turn up their noses at a man who prefers to learn from nature herself rather than from authors who are only her clerks."[1] This latter remark is similar in its reference to the epithet of Galileo, applied to men whose knowledge comes wholly from books and not from observation; namely, "paper philosophers."

*Bernardinus Telesius* (1508–1588). — His work, entitled *De Rerum Natura*, anticipated, in some degree at least, the *Novum Organon* of Bacon. Bacon himself says of him: "We think well concerning Telesius, and acknowledge him as a lover of truth, a useful contributor to science, an amender of some tenets, the first of recent men." Telesius set for himself a high aim and purpose, but in the application of his method he was not so fortunate, being dominated in his researches by speculation rather than the results of experimental inquiry. As to his professed method, he announces in the title of his *De Natura* that "the construction of the world, the magnitude and nature of the bodies contained in it, are not to be investigated by reasoning, which was done by the ancients, but are to be apprehended by the senses and collected from the things themselves." And in the Proem of the same work he says in the same strain that "they who before us have inquired concerning the construction of this world, and of the things which it contains, seem indeed to have prosecuted their examination with protracted vigils and great labor,

[1] Whewell, *Philosophy of the Inductive Sciences*, Vol. II. p. 369.

but never to have looked at it. For, as it were, attempting to rival God in wisdom, and venturing to seek for the principles and causes of the world by the light of their own reason, and thinking they had found what they had only invented, they made an arbitrary world of their own. We then, not relying on ourselves, and of a duller intellect than they, propose to ourselves to turn our regards to the world itself and its parts."

Following Telesius, and of his school, was Thomas Campanella (1568–1639). He was a contemporary of Bacon, and, under the influence of Telesius, early conceived the idea of an inductive method of research. At the age of twenty-two, he published a work whose character may be judged by its title, — "Thomas Campanella's Philosophy demonstrated to the senses, against those who have philosophized in an arbitrary and dogmatical manner, not taking nature for their guide; in which the errors of Aristotle and his followers are refuted from their own assertions and the laws of nature; and all the imaginations feigned in the place of nature by the Peripatetics are altogether rejected; with a true defence of Bernardin Telesius of Cosenza, the greatest of philosophers; confirmed by the opinions of the ancients, here elucidated and defended, especially those of the Platonists."

The ideas of Bacon, with their impetus to the inductive method of research, were not only anticipated by writers of books; but actual discoveries by zealous investigators were turning the attention of the thinking world to nature and her secrets.

There was an illustrious line of pioneers in this undiscovered country. There was Andrew Cæsalpinus (1520–1603), the founder of the science of botany; and earlier, Copernicus (1473–1543), advancing his heliocentric theory; and Gilbert (1540–1603), the court physician of Elizabeth and James, conducting with untiring perseverance his investigations of the nature of magnetism and electricity. Kepler, born ten years after Bacon, 1571, and Galileo, born in 1564, and their contemporary, Tycho Brahé, born in 1546, formed a triumvirate of scientific power and brilliancy, made resplendent by the glory of the heavens itself. It must be remembered, too, that at this time a new world had been discovered across the seas; the recent inventions of gunpowder, of the mariner's compass, and of the art of printing, all tended to stimulate the thought of the world, and usher in a new epoch in the history of civilization.

*Francis Bacon* (1561–1626). — Bacon's inductive system is given, for the most part, in the *Novum Organon*. The title of this work was in itself a protest against Aristotle and his logic, implying that Aristotle's *Organon* was now out of date and was to be superseded by the new. Bacon insists that all knowledge of nature has for its end the disclosing of the causes of things. According to the Aristotelian scheme, causes are formal, material, efficient, or final. Bacon is only concerned with the formal causes. For, he says, all events have their ground in the "forms" of things. By the form of a thing, he meant its essential nature. Where he

uses the form we may well supply the word law. To discover the forms of phenomena, it is necessary, according to Bacon, to collect as many instances as possible in which the phenomenon under investigation appears; together they form a *tabula præsentiæ.* In like manner, the instances in which the phenomenon is lacking are grouped in a *tabula absentiæ;* and a third group must be formed, — a *tabula graduum* in which the variations of intensity in the phenomena are compared with the varying intensity of other phenomena. The problem is then to be solved by a process of exclusion (*exclusio*); that is, the rejection or exclusion of the several qualities which are not found in some instance where the given quality is present, or are found in some instance where the given quality is absent, or are found to increase in some instance where the given quality decreases, or to decrease when the given quality increases. By this process an indication will be given by which an hypothesis may be framed, and finally verified by subsequent observation and experiment. In the sketch of this method it will be seen that his three tables of instances closely resemble the methods of agreement, of difference, and of concomitant variations. They, however, lack the precision of the later formulation of these methods. There is no hint at a systematic selection and variation of the instances; and no requirement, as in the method of difference, that two instances shall be so experimentally determined that they will agree in every point save the given phenomenon, which is present in the one

and absent from the other. Bacon, however, made a substantial contribution to the method of induction in general, in insisting upon the examination of instances themselves, and in ascending from them quite gradually the scale of the more general up to the most general, and in this he entered a vigorous protest against hasty generalization.

As to the manner of certifying the hypothesis formed after the process of collecting and sifting instances, Bacon has no recourse to deduction based upon the hypothesis and consequent verification. He seems to despise mathematical method as an ally of inductive inquiry; and, therefore, has no place in his scheme for the prediction of new phenomena by means of calculation. Of his nine divisions of aids to induction, he completed only the first, — Prerogative Instances. Of the instances which he enumerates, twenty-seven in all, only a few have any bearing directly upon the inductive method proper. Two sets of these instances may be considered as a crude statement of the methods of agreement and difference; the Solitary Instances, which either exhibit a phenomenon without any of its usual accompaniments or which agree in everything except some particular phenomenon, and Migratory Instances, where qualities are produced in bodies by evident causes, as, for instance, the producing of whiteness by pounding glass, also by stirring water into froth. These instances, however, as exhibited by Bacon, lack precision and the possibilities of accurate determination of causal connections. The only other group of instances hav-

ing special inductive significance is that of the *Instantia Crucis;* as before mentioned, such instances are valuable in deciding between rival hypotheses. With all the deficiencies of Bacon's method, however, his service to the thinking world is indisputable, in emphasizing the need of investigating phenomena themselves as a corrective of fanciful speculations, and in his vigorous warnings against prejudice, against intellectual indolence, against subjection of the mind to the trammels of authority, and against over-hasty generalizations.

*Locke* (1632–1704). — Locke applied the method of Bacon to the objects of inner experience. He declared that the data of all knowledge come from sensation, or sense-perception, and from reflection, and that there are no "innate ideas," and therefore no starting-point for *a priori* speculations. The method that had been found useful in actual discoveries, such as those of Newton, Kepler, and others, Locke insisted would prove productive also of rich results in the intellectual sphere.

*Isaac Newton* (1642–1727). — Scientific method was gradually formulating itself in the actual pursuits of scientific investigation, — not thought out as much as worked out, and its efficiency tested and confirmed by results. Newton gives form to that which was a result of many experiments, and of a mass of various experiences, in his Rules of Philosophizing (*Regulæ Philosophandi*) prefixed to the *Principia*.

These rules are as follows: —

1. The first rule is twofold: —

*a.* "Only real causes are to be admitted in explanation of phenomena."

*b.* "No more causes are to be admitted than such as suffice to explain the phenomena."

2. "In as far as possible, the same causes are to be assigned for the same kind of natural effects."

3. "Qualities that can neither be increased nor diminished in intensity, and that obtain in all bodies accessible to experiment, must be considered qualities of all bodies whatsoever."

4. "In philosophical experiment, propositions collected from phenomena by induction are to be held, notwithstanding contrary hypotheses, as either exactly or approximately true, until other phenomena occur whereby they are either rendered more exact or are proved liable to exceptions."

Newton's celebrated saying, "Hypotheses non fingo," was originally a protest against the supposition of the existence of occult or imaginary causes to explain phenomena, notably the Cartesian explanation of the celestial movements by vortices. Hypotheses of a different nature, and rationally grounded of course, did not fall under Newton's reprehension.

*Sir John Herschel* (1792–1871). — Herschel's *Discourse on the Study of Natural Philosophy* was published in 1832. John Stuart Mill reviewed this book in the *Examiner* and was evidently impressed and influenced by it. Herschel's design was to make the methods of science more explicit. These are contained in nine "propositions readily applicable to particular cases, or rules of philosophizing."

Of these propositions, the second, seventh, eighth, and ninth present substantially the experimental methods as afterwards more precisely formulated by Mill. These methods, however, he regards simply as means to discovery, and not methods of proof. Of the remaining propositions, the first is a more precise statement of Bacon's principle of exclusion, and is the foundation of the joint method of agreement and difference. The third proposition is that "we are not to deny the existence of a cause in favor of which we have a unanimous agreement of strong analogies, though it may not be apparent how such a cause can produce the effect, or even though it may be difficult to conceive its existence under the circumstances of the case." The fourth is that "contrary or opposing facts are equally instructive for the discovery of causes with favorable ones." The fifth recommends the "tabulation of facts in the order of intensity in which some peculiar quality subsists." The sixth rule insists upon the investigator keeping prominently in mind the possibility that "counteracting or modifying causes may subsist unperceived," and that this fact may be the means of explaining many apparent exceptions.

Herschel also emphasizes the necessity of combining induction and deduction in complicated inquiries; and, further, he explains the nature of empirical laws, as also the nature and tests of hypotheses. We can now see that the body of inductive principles begins at length to assume final form and proportion.

*Whewell* (1795–1866). — Dr. Whewell published his *Philosophy of the Inductive Sciences* in 1840, containing his system of induction. His method involves two principal processes, — the colligation of facts and the explication of conceptions. The investigator is to gather all the facts at his disposal, and upon them he is to superinduce a conception which will unify them, or colligate them. He says these conceptions are supplied by the mind, while facts are supplied by the sense. This, however, is a distinction that separates so widely the spheres of the particular facts, and the general conceptions, that upon such a basis a union of the two as explaining one by the other would be artificial and with no corresponding bond of reality. The colligating conception does not exist in the mind before or apart from its existence in fact. The attempt to fit facts to ready-made conceptions is of the nature of guess-work. Kepler's nineteen guesses regarding planetary orbits is an instance of attempting to superinduce conceptions upon a mass of facts. It is not a truly scientific or logical procedure, and the great danger of applying it lies in the fact that the mind all too readily tends to mould facts into the forms of prior conceptions.

"The Methods employed in the Formation of Science," the title of his concluding chapter, are three, as follows: Methods of Observation, Methods of Obtaining Clear Ideas, and Methods of Induction. The last principally concerns our present purposes. The methods of induction are
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methods of discovery rather than proof, save the

last, which is one of the experimental methods. They are the method of curves to express graphically the graduated results of several observations; the method of means, and the method of least squares, both designed to eliminate accidental accompaniments of constant causes by striking an average of several observations; and the method of residues. Whewell's method may be characterized in brief as a method of discovery rather than proof.

*John Stuart Mill* (1806–1873). — Mill's *Logic*, published in 1843, was essentially a method of proof rather than a method of discovery. His aim in formulating the methods in vogue in experimental science, was to discover the precise modes of their operation in order to apply the same in investigating the various mental, moral, social, and political phenomena. Bacon in the *Novum Organon* had asserted that this inductive method was applicable to the intellectual and moral sciences. This was no doubt suggestive to Mill, as it had been to Locke. Whately's *Logic*, published in 1827, influenced Mill,

see also minto

and was the means of turning his attention to logical studies. Whately's book was reviewed by Mill, when only twenty-one, in the *Westminster Review*. The revival in logical interest at this time and the departure from scholastic traditions have been traced to the influence of Edward Copleston, tutor at Oxford, and afterwards Bishop of Llandaff. Whately's work represented the first-fruits, and Mill's the richer and riper product of this revival of logic. It is a matter of more than passing interest to note that one of Whately's most active

collaborators in the work was John Henry Newman, so that, as Professor Minto says, "the common room of Oriel, which Mr. Froude describes as the centre from which emanated the High Church Movement, may also be said to have been the centre from which emanated the movement that culminated in the revolution of logic."

see also Minto

Mill's special office as regards induction consists in his crystallizing the principles and practices of the scientific investigators who had caught and reflected the spirit of modern research. The formulated methods of inductive logic, substantially as given by Mill, have become the recognized methods of all investigation that is actuated by a scholarly spirit and a scientific habit. Mill's contributions to the inductive logic have been so largely drawn from and so frequently referred to in the composition of this book, as to need no further comment here. The works of the more recent writers, as Lotze, Sigwart, Bosanquet, Jevons, Venn, etc., have also been noticed in the body of the text. Their work is largely critical, and no distinct inductive system is especially associated with any of their names.

# CHAPTER XX

## Logical Exercises

In the following examples, indicate the nature of the inferences, the methods employed, and the character of the results obtained, whether valid or invalid, and the reasons for the same.

1. In all unhealthy countries the greatest risk of fever is run by sleeping on shore. Is this owing to the state of the body during sleep, or to a greater abundance of miasma at such times? It appears certain that those who stay on board a vessel, though anchored at only a short distance from the coast, generally suffer less than those actually on shore. — Darwin in *Voyage of Naturalist.*

2. That the period of the tide should be accidentally the same as that of the culmination of the moon, that the period of the highest tide should be accidentally the same as that of the syzygies, is possible *in abstracto;* but it is in the highest degree improbable; the far more probable assumption is, either that sun and moon produce the tide, or that their motion is due to the same grounds as the motion of the tide.

3. In measuring the velocity of sound by experiments conducted at night with cannon, the results

at one station were never found to agree exactly with those at the other. Moreover, it was noticed that on the nights when the discordance was greatest, a strong wind was blowing nearly from one station to the other.

4. M. Melloni, observing that the maximum point of heat is transferred farther and farther towards the red end of the spectrum, according as the substance of the prism is more and more permeable to heat, inferred that a prism of rock-salt, which possesses a greater power of transmitting the calorific rays than any known body, ought to throw the point of greatest heat to a considerable distance beyond the visible part of the spectrum; and his prediction was verified by subsequent experiment.

5. During the middle of the eighteenth century Bonnet and Spallanzani discovered that the horns, tails, legs, eyes, or even head of some creatures, if cut off, would grow again. The tail and legs of a salamander were removed and reproduced themselves eight times in succession. By means of a number of experiments it has been found that the more simple the structure of an animal is, the more do its several parts possess a power of independent existence, and that in the more complex animals, the derangement of one part much more affects the action of the entire organism.

6. Professor Jevons has observed that economic crises have occurred at regular intervals of about ten years. This ten-year periodicity, moreover, seems to correspond to a similar periodicity of bad harvests;

and the cause of this seems to be a decennial periodicity in the spots on the sun.

7. What is the significance of the remark of Chevreul, the French scientist: "Every fact is an abstraction."

8. Also of the following remark of M. Espinas: "If human activity was incompatible with the order of things, the act of boiling an egg would have to be regarded as a miracle."

9. It had long been known that grasshoppers and crickets have on their anterior legs two peculiar, glassy, generally more or less oval, drum-like structures; but these were supposed by the older entomologists to serve as resonators, and to reinforce or intensify the well-known chirping sounds which they produce. Johannes Müller was the first who suggested that these drums or tympana act like the tympanum of our own ears, and that they are really the external parts of a true auditory apparatus. That any animal should have its ears in its legs sounds, no doubt, *a priori*, very unlikely, and hence probably the true function of this organ was so long unsuspected. — SIR JOHN LUBBOCK.

10. In simple fracture of the ribs, if the lung be punctured by a fragment, the blood effused into the pleural cavity, though freely mixed with air, undergoes no decomposition. Why air introduced into the pleural cavity through a wounded lung should have such wholly different effects from that entering directly through a wound in the chest was to me a complete mystery until I heard of the germ-

theory of putrefaction, when it at once occurred to me that it was only natural that air should be filtered of germs by the air-passages, one of whose offices is to arrest inhaled particles of dust and prevent them from entering the air-cells. — PROFESSOR LISTER.

11. If the lungs be emptied as perfectly as possible and a handful of cotton-wool be placed against the mouth and nostrils, and you inhale through it, it will be found on expiring this air through a glass tube that its freedom from floating matter is manifest. The application of this is obvious; if a physician wishes to hold back from the lungs of his patient, or from his own, the germs, or virus by which contagious disease is propagated, he will employ a cotton-wool respirator. — PROFESSOR TYNDALL.

12. In the desert of North Africa, where neither trees, brushwood, nor even undulation of the surface afford the slightest protection to its foes, a modification of color in animals which shall be assimilated to that of the surrounding country is absolutely necessary. Hence, without exception, the upper plumage of every bird, whether lark, chat, sylvian, or sand-grouse, and also the fur of all the smaller mammals and the skin of all snakes and lizards, is of one uniform isabelline, or sand color. — WALLACE.

13. Darwin, in investigating the difference in weight between cross and self fertilized plants, found that the six finest crossed plants averaged 108.16 ounces, whilst the six finest self fertilized plants averaged only 23.7 ounces or as 100 to 22.

14. Bees incessantly visit the flowers of the com-

mon broom and these are adapted by a curious mechanism for cross-fertilization. When a bee lights on the wing-petals of a young flower, it is slightly opened, and the short stamens spring out, which rub their pollen against the abdomen of the bee. If a rather older flower is visited for the first time (or if the bee exerts great force on a younger flower), the keel opens along its whole length, and the longer as well as the shorter stamens, together with the much elongated curved pistil, spring forth with violence. The flattened spoon-like extremity of the pistil rests for a time on the back of the bee, and leaves on it the load of pollen with which it is charged. As soon as the bee flies away, the pistil instantly curls round, so that the stigmatic surface is now upturned and occupies a position, in which it would be rubbed against the abdomen of another bee visiting the same flower. Thus, when the pistil first escapes from the keel, the stigma is rubbed against the back of the bee, dusted with pollen from the longer stamens, either of the same or another flower; and afterwards against the lower surface of the bee, dusted with pollen from the shorter stamens, which is often shed a day or two before that from the longer stamens. If the visits of bees are prevented, and if the flowers are not dashed by the wind against any object, the keel never opens, so that the stamens and pistil remain enclosed. Plants thus protected yield very few pods in comparison with those produced by neighboring uncovered bushes, and sometimes none at all. — DARWIN.

15. Baron Zach received a letter from Pons, a successful finder of comets, complaining that for a certain period he had found no comets, though he had sought diligently. Zach, a man of much sly humor, told him that no spots had been seen on the sun for about the same time — which was true — and assured him that when the spots came back, the comets would come with them. Some time after that he got a letter from Pons, who informed him, with great satisfaction, that he was quite right, that very large spots had appeared on the sun, and that he had found a fine comet shortly after. — DE MORGAN'S *Budget of Paradoxes.*

16. If Tellus winged be,
The earth a motion round;
Then much deceived are they
Who nere before it found.
Solomon was the wisest,
His wit nere this attained;
Cease, then, Copernicus,
Thy hypothesis vain!
— SYLVANUS MORGAN, 1652.

17. Weather Forecaster Dunn has prepared a chart showing the number of deaths from grip in New York City during the period from March 22 to May 16, 1891, establishing the relation between the death-rates and weather conditions during the grip epidemic of that year. Mr. Dunn has made a careful study of records of the disease, and selected the epidemic of 1891 as being the time when the grip was most pronounced.

He has apparently demonstrated that the weather is an important factor in the mortality of grip cases. He says that humidity or moisture in the air seems to be the most important element in causing the disease to spread. There is a corresponding increase of deaths with increasing humidity.

The fatality is most marked when the humidity is at its maximum and there is a sudden fall of the temperature. This is shown by the record of April 21, when the death-rate from grip was the highest ever known. During the twenty-four hours of that day 250 deaths were reported. On April 1 and April 30 the death-rate was also high. These were days following a sudden fall in temperature.

All through the epidemic the charts show an increasing death-rate with high or increasing humidity. The higher the humidity and the more sudden the fall in temperature, the greater was the number of deaths. When the temperature and the humidity dropped at the same time, there was a decrease in the death-rate, as Mr. Dunn points out by several examples. He says that the lesson to be learned from his chart is that those suffering from an incipient attack of the grip should be most cautious of the cold, humid days that immediately follow the warm, damp ones.

18. If in a reservoir immersed in water, the air be compressed to the extent of ten atmospheres, and supposing that now, when the compressed air has acquired the temperature of the water, it be allowed to act upon a piston loaded by a weight, the weight is raised. At the same time the water

becomes cooler, showing that a certain quantity of heat had disappeared in producing the mechanical effort of raising the weight.

19. That the feeling of effort is largely, if not entirely, of peripheral, rather than central origin, appears from such experiments as the following: —

Hold the finger as if to pull the trigger of a pistol. Think vigorously of bending the finger, but do not bend it. An unmistakable feeling of effort results. Repeat the experiment, and notice that the breath is involuntarily held, and that there are tensions in the other muscles. Repeat the experiment again, taking care to keep the breathing regular and the other muscles passive. Little or no feeling of effort will now accompany the imaginary bending of the finger. — Ferrier.

20. As to the nature of petrified shells, Quirini conceived that as earthy particles united in the sea so as to form the shells of Mollusca, the same crystallizing process might be effected on the land; and that in the latter case, the germs of the animals might have been disseminated through the substance of the rocks, and afterwards developed by virtue of humidity.

21. Voltaire suggested that the marine shells found on the tops of mountains are Eastern species dropped from the hats of pilgrims as they returned from the Holy Land.

22. The epicyclical theory of the heavens was confirmed by its predicting eclipses of the sun and moon, configurations of the planets, and other celestial phenomena.

23. Arfvedson discovered lithia, by perceiving an excess of weight in the sulphate produced from a small portion of what he considered as magnesia present in a mineral he had analyzed.

24. We see among the nebulæ (which are diffused along the Milky Way) instances of all degrees of condensation, from the most loosely diffused fluid, to that separation and solidification of parts by which suns and satellites and planets are formed; and thus we have before us instances of systems in all their stages; as in a forest we see trees in every period of growth. — LAPLACE.

25. It had been deductively inferred from the Copernican theory that the planets, Venus and Mercury, ought to pass through phases, like the moon, and the telescope revealed this to be the case.

26. Werner, says Sir Charles Lyell, had not travelled to distant countries; he had merely explored a small portion of Germany, and conceived, and persuaded others to believe, that the whole surface of our planet, and all the mountain chains in the world, were made after the model of his own province.

27. Scheiner was a monk; and on communicating to the superior of his order the account of the spots on the sun, received the reply: "I have searched through Aristotle, and can find nothing of the kind mentioned: be assured, therefore, that it is a deception of your senses, or of your glasses."

28. When we are told that a man has become deranged from anxiety or grief, we have learned very little if we rest content with that. How does

it happen that another man, subjected to an exactly similar cause of grief, does not go mad?—MAUDSLEY.

29. It was a general belief at St. Kilda that the arrival of a ship gave all the inhabitants colds. Dr. John Campbell took pains to ascertain the fact and to explain it as the effect of effluvia arising from human bodies; it was discovered, however, that the situation of St. Kilda renders a northeast wind indispensably necessary before a ship can make the landing.

30. Chrysippus maintained that cock-fighting was the final cause of cocks, these birds being made by Providence in order to inspire us by the example of their courage.

31. Touch in succession various objects on the table. A paper-weight, if metallic, is usually cold to the touch; books, paper, and especially a woollen table-cover, comparatively warm. Test them by means of a thermometer, and there will be little or no difference in their temperatures. Why then do some feel cold, others warm to the touch? The sense of touch does not inform us directly of temperature, but of the rate at which our finger gains or loses heat. As a rule, bodies in a room are colder than the hand, and heat always tends to pass from a warmer to a colder body. Of a number of bodies, all equally colder than the hand, that one will seem coldest to the touch, as the metallic, which is able most rapidly to convey away heat from the hand. —TAIT.

32. One of Joule's experiments concerning the

mechanical value of light is as follows: He compared the heat evolved in the wire conducting a galvanic current when the wire was ignited by the passage of the current, with that evolved when with an equal current it was kept cool by immersion in water. These experiments showed a small but unmistakable diminution of the heat when light also was given out. — TAIT.

33. It is an illusion in psychology and a corruption of logic to take the conditions which occasion the logical operations of thought for the operations themselves. There is only one delusion more desperate still, — to imagine that a complete physical theory of the nervous system will explain that which is itself the condition of any theory being possible at all. — LOTZE.

34. During the retreat of the Ten Thousand a cutting north wind blew in the faces of the soldiers; sacrifices were offered to Boreas, and the severity of the wind immediately ceased, which seemed a proof of the god's causation.

35. It has been shown by observation that overdriven cattle, if killed before recovery from their fatigue, become rigid, and putrefy in a surprisingly short time. A similar fact has been observed in the case of animals hunted to death, cocks killed during a fight, and soldiers slain in battle. The contrary is remarked when the muscular exercise has not been great or excessive.

36. A correct analysis of lapis lazuli was suspected to be erroneous because there seemed to be nothing in the elements assigned to it, which were

silica, alumina, soda, sulphur, and a trace of iron, to account for the brilliant blue color of the stone.

37. According to the theory that the earth has but a thin crust, it is still substantially a liquid globe, and therefore, under the attractive influence of the sun and moon, it ought to behave like a yielding liquid. According to Hopkins, Thomson, and others, the earth in all its astronomical relations behaves like a rigid solid, — a solid more rigid than a solid globe of glass, — and the difference between the behavior of a liquid globe and a solid globe could easily be detected by astronomical phenomena. — Le Conte.

38. Many years ago I was struck with the fact that humblebees, as a general rule, perforate flowers only when these grow in large numbers near together. In a garden where there were some very large beds of *Stachys coccinea* and of *Pentstemon argutus*, every single flower was perforated; but I found two plants of the former species growing quite separate with their petals much scratched, showing that they had been frequently visited by bees, and yet not a single flower was perforated. I found also a separate plant of the Pentstemon, and saw bees entering the mouth of the corolla and not a single flower had been perforated. In the following year (1842) I visited the same garden several times: on the 19th of July humblebees were sucking the flowers in the proper manner, and none of the corollas were perforated. On the 7th of August all the flowers were perforated, even those on some few plants of the salvia, which grew at a little distance from the great

bed. On the 21st of August only a few flowers on the summits of the spikes of both species remained fresh, and not one of these was now bored. Again, in my own garden every plant in several rows of the common bean had many flowers perforated; but I found three plants in separate parts of the garden which had sprung up accidentally, and these had not a single flower perforated. General Strachey formerly saw many perforated flowers in a garden in the Himalaya, and he wrote to the owner to inquire whether this relation between the plants growing crowded and their perforation by bees there held good, and was answered in the affirmative. Hence it follows that the red clover and the common bean when cultivated in great masses in fields, *Erica tetralix* growing in large numbers on heaths, —rows of the scarlet kidney-bean in the kitchen garden, — and masses of any species in the flower garden are all eminently liable to be perforated. The explanation of this is not difficult. Flowers growing in large numbers attract crowds of insects. They are thus stimulated to work quickly by rivalry. Also many flowers have their nectaries dry, which is most quickly discovered by biting holes in them. — CHARLES DARWIN.

39. The seat of sensation is in the heart, as it is in the centre of the body; the brain is cold in order that it may counteract the heat of the heart. In order to temper the coldness of the brain, blood is conveyed to the membrane which envelops it by means of veins or channels. But, lest the heat so conveyed should injure the brain, the veins, instead of being

large and few, are small and many, and the blood conveyed, instead of being copious and thick, is thin and pure. — ARISTOTLE.

40. The lungs of a fox must be a specific for asthma, because that animal is remarkable for its strong powers of respiration. — PARIS' *Pharmacologia.*

41. Galileo discovered, by the use of his telescope, the four small satellites which circulate round Jupiter. It was then inferred that what happened on the smaller scale might also be found true of the larger planetary system.

42. The first step toward the discovery of photography was the knowledge that visual light caused a chemical change in iodide of silver. The second step was to fix in permanent position the portion of the substance changed by the light, while the unchanged portion was removed.

From what is known of the chemical elements and their compounds, it seems highly probable that numerous compounds may exist which are sensitive in the same way to waves of entirely different lengths from those that produce vision. Even with the salts of silver it has long been known that the range of wave-lengths capable of producing photographic effect is much greater than the visual range; and that the wave-lengths which produce the maximum physiological effect (light) are not the same as those that produce the maximum photographic effect.

It has been shown by Professor S. R. Langley that flint glass is transparent to waves about four

times as long as the longest in the visual range; and that rock-salt is transparent to a range below the red end of the visible spectrum twenty-nine times as long as the entire visual range. Glass is opaque to very short waves, its limit in that direction being nearly coincident with the visual limit. Quartz, on the other hand, is transparent to a range of short waves extending far beyond the visual limit, but is opaque to very short waves. May not these substances prove valuable in this new field of actinography, as quartz trains have proved in photographing the ultra-violet spectrum?

Should the report of this discovery (Röntgen's) be confirmed, we cannot fail to accord the highest praise to this new triumph of science, and to predict a development of the new field of actinography that may prove of greater importance than photography.

From the analogy between this form of radiant energy and dark heat it might appropriately be called "dark light." — *The Electrical World.*

43. As to the theory of geyser-eruption, the following principles have been established. The boiling-point of water rises as the pressure increases, being 293° for a pressure of four atmospheres. Also, if the pressure be diminished when the water is under very strong pressure, the water will immediately flash into steam. Moreover, if the circulation is impeded, as when the water is contained in long, narrow, irregular tubes, and heated with great rapidity, the boiling-point will be reached below while it is far from this point in

the upper part of the tube. Therefore at the moment of eruption, the boiling-point for the lowest depth is actually reached. The water there being transferred into steam, the expanding steam would lift the whole column of water in the tube, causing an overflow. This would diminish the pressure in every part of the tube, and consequently a large quantity of water before very near the boiling-point would flash into steam and instantly eject the whole of the water in the pipe, the steam rushing out immediately afterwards. The premonitory cannonading beneath is evidently produced by the collapse of large steam-bubbles rising through the cooler part of the water of the tube. — BUNSEN'S THEORY.

44. Mackenzie's theory of geyser-eruption is that the geyser pipe is connected by a narrow conduit with the lower part of a subterranean cave, whose walls are heated by the near vicinity of volcanic fires. The water rising above the opening of the conduit, and changing into steam, and having no way of escape, would, through pressure thus caused, be forced up the pipe, and the steam rushing after it. Professor Le Conte says of this theory: If there were but one geyser, this would be considered a very ingenious and probable hypothesis; for we may conceive of a cave and a conduit so constructed as to account for the phenomena. But there are so many geysers, that it is inconceivable that all of them should have caves and conduits so peculiarly constructed. This theory, therefore, is entirely untenable.

45. It has been found by experiment that a current moving at the rate of three inches per second will take up and carry along fine clay; moving six inches per second, will carry fine sand; eight inches per second, coarse sand the size of linseed; twelve inches, gravel; twenty-four inches, pebbles; three feet, angular stones of the size of a hen's egg. It will be readily seen that the carrying power increases much more rapidly than the velocity. For instance, a current of twelve inches per second carries gravel, while a current of three feet per second, only three times greater velocity, carries stones many hundred times as large as grains of gravel.

46. If wood be soaked in a strong solution of sulphate of iron (copperas) and dried, and the same process be repeated until the wood is highly charged with this salt, and then burned, the structure of the wood will be preserved in the peroxide of iron left. Also, it is well known that the smallest fissures and cavities in rocks are speedily filled by infiltrating waters with mineral matters. Now, wood buried in soil soaked with some petrifying material becomes highly charged with the same, and the cells filled with infiltrated matter, and when the wood decays the petrifying material is left, retaining the structure of the wood. In nature also there is an additional process, not illustrated by the experiment, or by the example of infiltrated fillings. As each particle of organic matter passes away by decay, a particle of mineral matter takes its place, until finally the whole of the organic matter is replaced.

47. As to the origin of bitumen, the following observations have been made: Certain organic matters at ordinary temperature, in presence of abundant moisture, and out of contact of air, will undergo a species of decomposition or fermentation by which an oily or tarry substance, similar to bitumen, is formed. In the interior of heaps of vegetable substance, such bituminous matter is often found. Fossil cavities have been found in solid limestone containing bitumen, evidently formed by decomposition of the animal matter. So, also, shales have been found in Scotland, filled with fishes which have changed into bitumen.

48. Count Rumford in 1798 proved that the common notion that heat was a substance was false, by boring a large piece of brass, under great pressure of the borer, whilst the brass was in a gallon of water; and at the end of two and one half hours the water actually boiled.

49. Kenelm Digby's treatment of wounds was to apply an ointment, not to the wound itself, but to the sword that had inflicted it, to dress this carefully at regular intervals, and in the meantime, having bound up the wound, to leave it alone for seven days. It was observed that many cures followed upon this treatment.

50. When Pascal's barometer was carried to the top of Puy-de-Dôme, and the mercury in it fell, it was inferred that the fall of the mercury was due to the change in elevation. Before finally accepting this conclusion, the barometer was placed in exposed positions and in sheltered, when the wind blew and

when it was calm, in rain and in fog; and these varying circumstances did not materially affect the result.

51. A French experimenter, Pouchet, thought he had obtained indubitable evidence of spontaneous generation. He took infusions of vegetable matter, boiled them to a pitch sufficient to destroy all germs of life, and hermetically sealed the liquid in glass flasks. After an interval, micro-organisms appeared. It seems that at a certain stage in Pouchet's process, he had occasion to dip the mouths of the flasks in mercury. It occurred to Pasteur, in repeating the experiments, that germs might have found their way in from the atmospheric dust on the surface of this mercury. And when he carefully cleansed the surface of the mercury, no life appeared afterwards in his flasks.

52. The causes to which the decay of the natives of New Zealand has been assigned are given as follows: drink, disease, European clothing, peace, and wealth. — *Journal of the Anthropological Institute.*

53. An eminent judge was in the habit of jocosely propounding, after dinner, a theory that the cause of the prevalence of Jacobinism was the practice of bearing three names. He quoted, on one side, Charles James Fox, Richard Brinsley Sheridan, John Horne Tooke, John Philpot Curran, Samuel Taylor Coleridge, Theobald Wolfe Tone. On the other hand there were, William Pitt, John Scott, William Windham, Samuel Horsley, Henry Dundas, Edmund Burke. Moreover, the practice of giving children three names has been a growing

practice, and Jacobinism has also been growing. The practice of giving children three names is more common in America than in England. In England, we still have a King and a House of Lords; but the Americans are Republicans. Burke and Theobald Wolfe Tone are both Irishmen; therefore the being an Irishman is not the cause of Jacobinism. Horsley and Horne Tooke are both clergymen; therefore the being a clergyman is not the cause of Jacobinism. Fox and Windham were both educated at Oxford; therefore the being educated at Oxford is not the cause of Jacobinism. Pitt and Horne Tooke were both educated at Cambridge; therefore the being educated at Cambridge is not the cause of Jacobinism. The cause is, therefore, the having three names. — MACAULAY.

54. The exotic Pelargonia have a peculiar herring-bone structure in the petals; moreover, the herring-bone structure is conjoined in the Pelargonia with the general characteristics of the Geranieæ. Also the flowers with such seed-vessels as our wild geraniums have the characters of Geranieæ. It is, therefore, exceedingly probable that our wild geraniums should have the peculiar herring-bone structure.

55. Colonies ought not to rebel against the mother country, since they are its children and children ought not to rebel against their parents.

56. Finding that the size of towns varies concomitantly with the size of the rivers on which they are built, an observer might infer that the size of the river was due to the size of the town.

57. An eminent author, writing on the work of

the English Church before the Tractarian movement, contrasts the newer state of things unfavorably with the older, because the Church in those former days taught us to use religion as a light by which to see our way along the road of duty. Without the sun our eyes would be of no use to us; but if we look at the sun, we are simply dazzled and can see neither it nor anything else. It is precisely the same with theological speculations. If the beacon lamp is shining, a man of healthy mind will not discuss the composition of the flame.

58. Scarlet color prevails among balsamina, Euphorbia, Pelargonium, poppy, Salvia, Bouvardia, and Verbena, yet none of the scarlets are of sweet perfumes. Some of the light-colored balsams and verbenas are sweet-scented, but none of the scarlets are. The common sage with blue blooms is odoriferous both in flower and foliage; but the scarlet salvias are devoid of smell. None of the sweet-scented-leaved Pelargoniums have scarlet blooms, and none of the scarlet bloomers have sweet scent of leaves nor of blooms. Some of the white-margined poppies have pleasant odors; but the British scarlets are not sweet-scented. The British white-blooming hawthorn is of the most delightful fragrance; the scarlet flower has no smell. Some of the honeysuckles are sweetly perfumed, but the scarlet trumpet is scentless.

59. The productive powers of plants, judging from the increased fertility of the parent-plants and from the increased powers of growth in the offspring, are favored by some degree of differen-

tiation in the elements which interact and unite so as to form a new being. Here we have some analogy with chemical affinity or attraction, which comes into play only between atoms or molecules of a different nature. As Professor Miller remarks: "Generally speaking, the greater the difference in the properties of two bodies, the more intense is their tendency to mutual chemical action. But between bodies of a similar character the tendency to unite is feeble."

60. In affirming that the growth of the body is mechanical, and that thought, as exercised by us, has its correlative in the physics of the brain, I think the position of the "materialist" is stated, as far as that position is a tenable one. I think the materialist will be able finally to maintain this position against all attacks; but I do not think, in the present condition of the human mind, that he can pass beyond this position. I do not think he is entitled to say that his molecular groupings and his molecular motions explain everything. In reality, they explain nothing. The utmost he can affirm is the association of the two classes of phenomena, of whose real bond of union he is in absolute ignorance. The problem of the connection of body and soul is as insoluble in its modern form as it was in the prescientific ages. Phosphorus is known to enter into the composition of the human brain, and a trenchant German writer has exclaimed, "Ohne Phosphor, kein Gedanke!" That may or may not be the case; but even if we knew it to be the case, the knowledge would not lighten our darkness. — Tyndall.

61. Granting that Hegel was more or less successful in constructing, *a priori*, the leading results of the moral sciences, still it was no proof of the correctness of the hypothesis of identity, with which he started. The facts of nature would have been the crucial test. That in the moral sciences traces of the activity of the human intellect and of the several stages of its development should present themselves, was a matter of course; but surely, if nature really reflected the result of the thought of a creative mind, the system ought, without difficulty, to find a place for her comparatively simple phenomena and processes. — HELMHOLTZ.

62. When young Galileo was a student at Pisa, he noticed one day, during the service at the great Cathedral, the chandelier swinging backwards and forwards, and convinced himself, by counting his pulse, that the duration of the oscillations was independent of the arc through which it moved.

63. Goethe enunciated the existence of a resemblance between the different parts of one and the same organic being. According to Goethe's own account, the idea first occurred to him while looking at a fan-palm at Padua. He was struck by the immense variety of changes of form which the successively developed stem-leaves exhibit, by the way in which the first simple root leaflets are replaced by a series of more and more divided leaves, till we come to the most complicated. He afterwards succeeded in discovering the transformation of stem-leaves into sepals and petals, and of sepals and petals into stamens, nectaries, and ovaries, and thus

he was led to the doctrine of the metamorphosis of plants which he published in 1790.

64. A fortunate glance at a broken sheep's-skull, which Goethe found by accident on the sand of the Lido at Venice, suggested to him that the skull itself consisted of a series of very much altered vertebræ. At first sight no two things can be more unlike than the broad, uniform, cranial cavity of the mammalia, enclosed by smooth plates, and the narrow cylindrical tube of the spinal marrow, composed of short, massy, jagged bones. — HELMHOLTZ.

65. The existence of the so-called blind spot in the eye was first demonstrated by theoretical arguments. While the long controversy whether the perception of light resided in the retina or the choroid was still undecided, Mariotte asked himself what perception there was where the choroid is deficient. He made experiments to discover this point and in the course of them discovered the blind spot.

66. Haüy observed that crystals of "heavy spar" from Sicily and those from Derbyshire (which were considered to be the same substance) differed in their angles of cleavage by three and one-half degrees, and remarked: "I could not suppose that this difference was the effect of any law of decrement; for it would have been necessary to suppose so rapid and complex a law, that such a hypothesis might have been justly regarded as an abuse of the theory." Vauquelin by chemical analysis discovered that the base of the crystals from Sicily was strontia, and that of those from Derbyshire was baryta. These facts,

becoming known to Haüy, enabled him by inference to discover that the angles of crystals might be employed as a test for the presence of different substances which very nearly resemble each other in other respects.

67. Graebe, a German chemist, in investigating a class of compounds, called the quinones, determined incidentally the molecular structure of a body closely resembling alizarine, which had been discovered several years before. This body was derived from naphthaline, and, like many similar derivatives, was reduced back to naphthaline when heated with zinc-dust. This circumstance led the chemist to heat also madder alizarine with zinc-dust, when, to his surprise, he obtained anthracene. Of course, the inference was at once drawn that alizarine must have the same relation to anthracene that the allied coloring matter bore to naphthaline; and, more than this, it was also inferred that the same chemical processes which produced the coloring matter from naphthaline when applied to anthracene would yield alizarine. The result fully answered these expectations, and now alizarine is manufactured on a large scale from anthracene obtained from coal-tar. — COOKE, *The New Chemistry.*

68. Sir Charles Lyell, by studying the fact that the river Ganges yearly conveys to the ocean as much earth as would form sixty of the great pyramids of Egypt, was enabled to infer that the ordinary slow causes now in operation upon the earth would account for the immense geological changes

that have occurred, without having recourse to the less reasonable theory of sudden catastrophes.

69. Joule's experiments show that when heat is produced by the consumption of work, a definite quantity of work is required to produce that amount of heat which is known to the physicists as the unit of heat; the heat, that is to say, which is necessary to raise one gramme of water through one degree centigrade. The quantity of work necessary for this is, according to Joule's best experiments, equal to the work which a gramme would perform in falling through a height of 425 metres.

In order to show how closely concordant are his numbers, I will adduce the results of a few series of experiments which he obtained after introducing the latest improvements in his methods.

(1) A series of experiments in which water was heated by friction in a brass vessel. In the interior of this vessel a vertical axle provided with sixteen paddles was rotated, the eddies thus produced being broken by a series of projecting barriers, in which parts were cut out large enough for the paddles to pass through. The value of the equivalent was 424.9 metres.

(2) Two similar experiments, in which mercury in an iron vessel was substituted for water in a brass one, gave 425 and 426.3 metres respectively.

(3) Two series of experiments, in which a conical ring rubbed against another, both surrounded by mercury, gave 426.7 and 425.6 metres respectively.

Exactly the same relations between heat and

work were also found in the reverse process; that is, when work was produced by heat. — HELMHOLTZ.

70. A gas which is allowed to expand with moderate velocity becomes cooled. Joule was the first to show the reason of this cooling. For the gas has, in expanding, to overcome the resistance which the pressure of the atmosphere and the slowly yielding sides of the vessel oppose to it; or, if it cannot of itself overcome this resistance, it supports the arm of the observer, which does it. Gas thus performs work, and this work is produced at the cost of its heat. Hence the cooling. If, on the contrary, the gas is suddenly allowed to issue into a perfectly exhausted space where it finds no resistance, it does not become cool, as Joule has shown.— HELMHOLTZ.

71. The principal feature in the plan of my attempt to penetrate into the North Polar region, or if possible to cross it, is, in brief, to try to make use of the currents of the sea, instead of fighting against them. My opinion is, as I have already explained on several occasions, that there must somewhere run currents into the Polar region, which carry the floe-ice across the Polar Sea, first northward toward the Pole, and then southward again into the Atlantic Ocean. That these currents really exist all Arctic expeditions prove, as most of them have had to fight against the currents and against the ice drifting southward, because they have tried to get northward from the wrong side. I think a very simple conclusion must be drawn from this fact that currents and drifting ice are constantly coming from the unknown north, viz.: Currents and perhaps also

ice must pass into this same region, as the water running out must be replaced by water running in. This conclusion is based upon the simplest of all natural laws; but there seem to be people who will not even admit the necessity of this.

That such currents run across the North Polar region is also proved by many facts. I may mention the great quantities of Siberian driftwood which are annually carried to the shores of Spitzbergen and Greenland; it comes in such abundance and with such regularity that it is quite impossible it should be carried to these shores, so far from the original home, by occasional winds or currents. There must be a regular communication between the coasts of Siberia and those of Spitzbergen and Greenland. By this same communication were several objects from the unfortunate *Jeannette* carried to the Greenland coast. The *Jeannette* sank in June, 1881, to the north of the New Siberian Islands, and three years afterward, in June, 1884, a great many objects belonging to her or her crew were found on an ice-floe on the southwest coast of Greenland. This floe can only have been brought there by the same current which carries the driftwood. By this same current an Esquimau implement, a throwing-stick or harpoon-thrower, was also carried the long way from Alaska to the west coast of Greenland. There can, in my opinion, be no doubt of the existence of such a communication or current across the North Polar region from the Siberian side to the Greenland side. — Dr. Nansen in *The Strand Magazine.*

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